Calculation and Representation Policy

Norton CEVC Primary School

Written Calculation

The overall aim is that when children leave our schools they:

• have a secure knowledge of number facts and a good understanding of the four operations;

• are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers;

• make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;

• have an efficient, reliable, written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;

• use a calculator effectively, using their mental skills to monitor the process, check the steps involved and decide if the numbers displayed make sense.

Progression towards a standard written method of calculation

INTRODUCTION

In Norton we use structured and systematic approach to teaching number.

There is a considerable emphasis on teaching mental calculation strategies. Up to the age of 7

(Year 2) informal written recording should take place regularly and is an important part of learning and understanding. More formal written methods should follow only when the child is able to use a wide range of mental calculation strategies.

REASONS FOR USING WRITTEN METHODS:

- To aid mental calculation by writing down some of the numbers and answers involved
- To make clear a mental procedure for the pupil
- To help communicate methods and solutions
- To provide a record of work to be done
- To aid calculation when the problem is too difficult to be done mentally
- To develop and refine a set of rules for calculation

STANDARD VOCABULARY FOR EACH OPERATION

+	-
Get some more, real story, maths story, same value different appearance, tens, units, hundreds, thousands, place value, digit, value, combine, sum, total, add, addition, more, plus, increase, sum, total, altogether, score, double, near double, how many more to make…?, equals, sign, regroup, tens boundary, hundreds boundary, units boundary, tenths boundary, inverse, fair swap	Get ready to take away, real story, maths story subtract, take away, minus, decrease, leave, how many are left/left over? difference between, half, halve, how many more/fewer is.. than…?, how much more/less is…?, equals, sign, tens boundary, hundreds boundary, units boundary, tenths boundary, inverse, regroup, fair swap, same value different appearance
x	**÷**
Lots of, groups of, times, product, multiply, multiplied by, multiple of, once, twice, three times, four times, five times… ten times, repeated addition, array, row, column, double, regroup, fair swap, inverse, same value different appearance	Halve, share, share equally, one each, two each, three each…, group in pairs, threes… tens, equal groups of, divide, divided by, divided into, divisible by, remainder, factor, quotient, inverse, regroup, fair swap, inverse, same value different appearance

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary.

The calculation policy is divided into four sections: addition, subtraction, multiplication and division. At the start of each section, you will find an overview of the progression of skills. Calculations involving decimal numbers and fractions are included.

The calculation policy follows the same concrete, pictorial, abstract approach as our main schemes of learning. Where appropriate, sentence stems and key questions are included alongside the key representations.

Below details the representations and calculation policy we follow when teaching key calculation concepts. Not all representations will be used during a lesson; teachers use their professional judgement to select representations which best suit the needs of their class and follow through the sequence of lessons.

Progression of skills - Addition

Year group	Skill
Reception	• Conceptually subitise to 5 • 1 more • Notice the composition of numbers within 10 • Combine 2 groups • Add more
Year 1	• Add together • Add more • Bonds within 10 • Related facts within 20 • Missing numbers

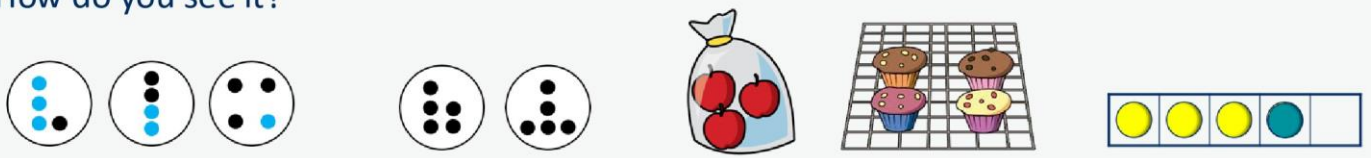
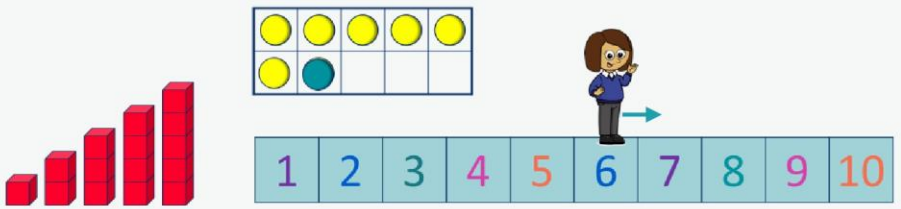
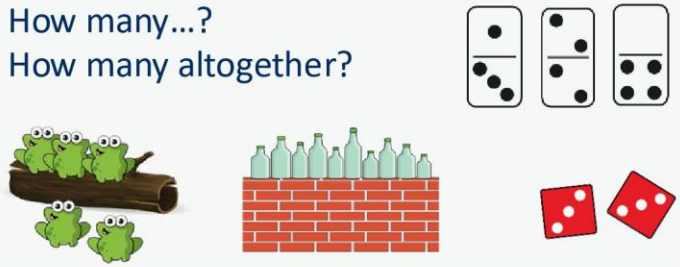
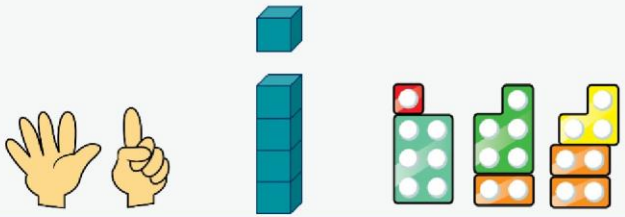
Progression of skills - Addition

Year group	Skill
Year 2	• Add 1s to any number (related facts) • Add three 1-digit numbers • Add across a 10 • Add multiples of 10 • Add 10s to any number • Add two 2-digit numbers (not across a ten) • Add two 2-digit numbers (across a ten) • Missing numbers
Year 3	• Add 1s, 10s and 100s to a 3-digit number • Add two numbers (no exchange) • Add two numbers across a 10 or 100 • Complements to 100 • Add fractions with the same denominator within 1 whole • Calculate the duration of events

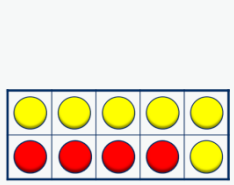
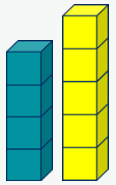
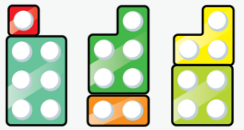
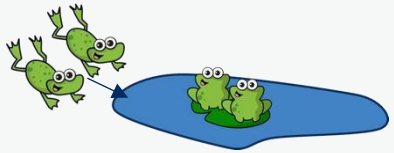
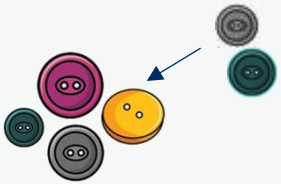
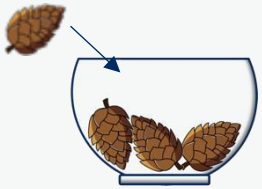
Progression of skills - Addition

Year group	Skill
Year 4	• Add 1s, 10s and 100s to a 4-digit number • Add up to two 4-digit numbers • Add decimal numbers in the context of money • Add fractions and mixed numbers with the same denominator beyond 1 whole
Year 5	• Add using mental strategies • Add whole numbers with more than 4 digits • Add decimals with up to 2 decimal places • Complements to 1 • Add fractions with denominators that are a multiple of one another
Year 6	• Add integers up to 10 million • Add decimals with up to 3 decimal places • Order of operations • Negative numbers • Add fractions

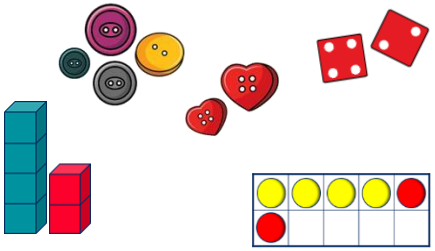
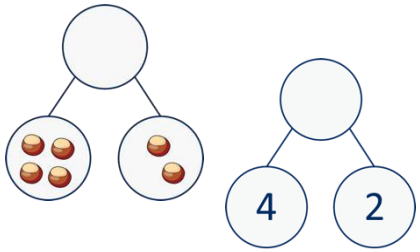
Addition

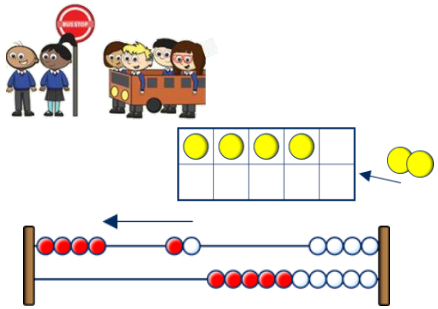
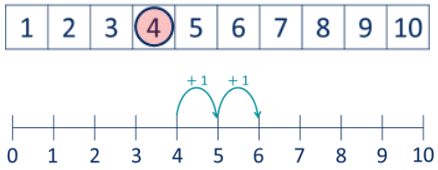
Reception	• Have a deep understanding of numbers to 10, including the composition of each number. • Subitise (recognise quantities without counting) up to 5 • Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.
Progression of skills	**Key representations**
Conceptually subitise to 5 Notice the parts that make up the whole.	What do you see? How do you see it? 
1 more Continue to link to stories, songs and rhymes.	1 more than … is … 
Notice the composition of numbers within 10 Link to stories, songs and rhymes.	How many…? How many…? How many altogether?  How many ways can you make…? 

Addition

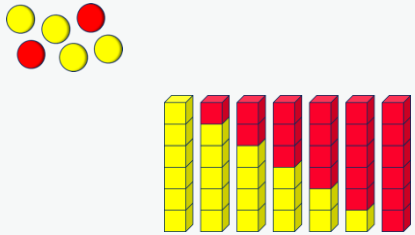
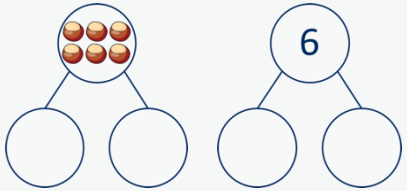
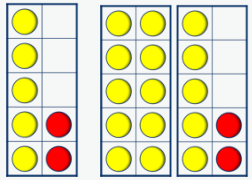
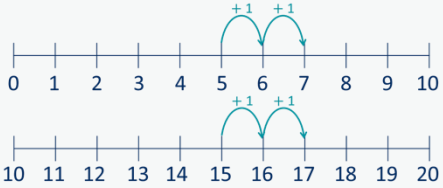
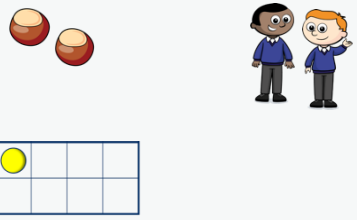
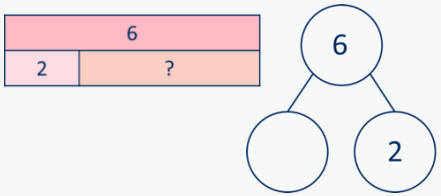
Progression of skills	Key representations	
Combine 2 groups 2 groups are combined to find the total.	There are …. There are …. There are …. altogether.   	…. and …. make ….   
Add more A quantity is increased.	First… Then…. Now….   	I have …. I add …. more. Now I have….   

Addition

Year 1	• Read, write and interpret mathematical statements involving addition (+) and equals (=) signs. • Represent and use number bonds within 20 • Add 1-digit and 2-digit numbers to 20, including zero. • Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as $7 = \square + 2$		
Progression of skills	**Key representations**		
Add together (aggregation) 2 quantities are combined to find the total.	There are … There are … There are … altogether. 	… is a part. … is a part. … is the whole. 	… plus … is equal to … … is equal to …. + … $4 + 2 = 6$ $2 + 4 = 6$ $6 = 4 + 2$ $6 = 2 + 4$

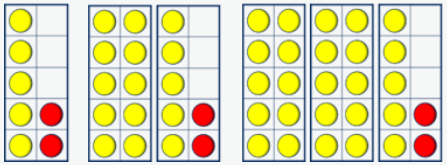
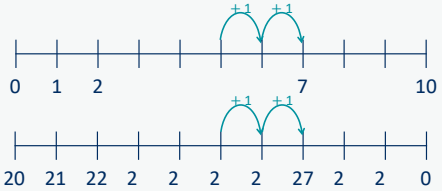
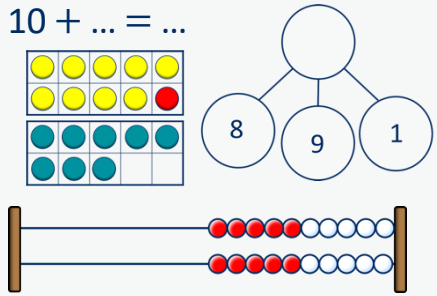
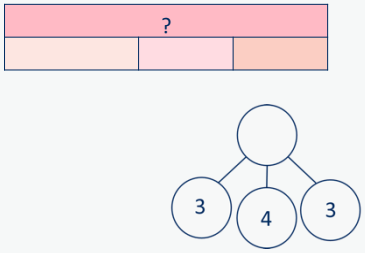
Add more (augmentation) A quantity is increased.	First… Then… Now… 	I start at … I jump on … I land on …  	… plus … is equal to … is equal to … + … $4 + 2 = 6$ $2 + 4 = 6$ $6 = 4 + 2$ $6 = 2 + 4$

Addition

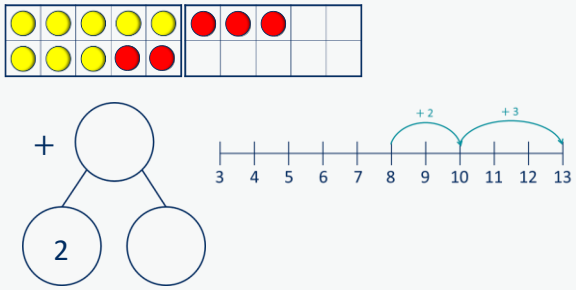
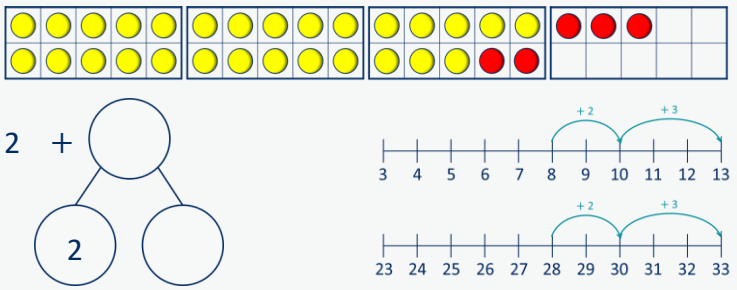
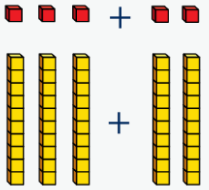
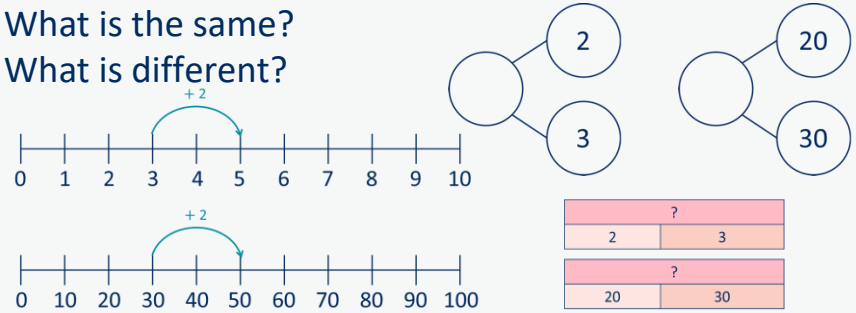
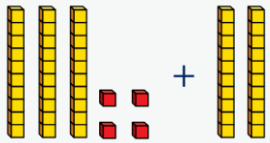
Progression of skills	Key representations		
Bonds within 10 Include bonds for each number within 10 Encourage children to notice patterns.	… is made of … and … … and … make … 	… can be partitioned into … and … 	… plus … is equal to … $6 + 0 = 6$ $5 + 1 = 6$ $4 + 2 = 6$ $3 + 3 = 6$ $2 + 4 = 6$ $1 + 5 = 6$ $0 + 6 = 6$
Related facts within 20 Make links to known facts.	I know that … and …= … so … and …= … 	… more than … is … so … more than … is … 	What patterns do you notice? $5 + 2 = 7$ $15 + 2 = 17$ $7 = 5 + 2$ $17 = 15 + 2$
Missing numbers Make links to known facts.	How many more do you need to make …? 	If … is the whole and … is a part, the other part must be… 	… plus … is equal to … $2 + \square = 6$ $6 = 2 + \square$ 

Addition

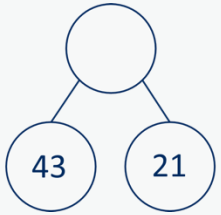
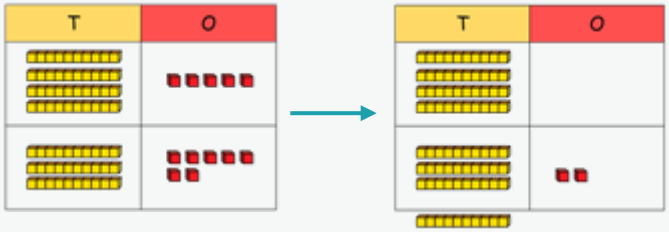
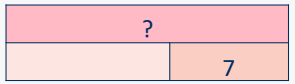
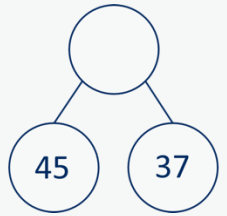
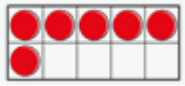
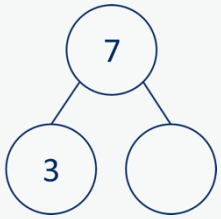
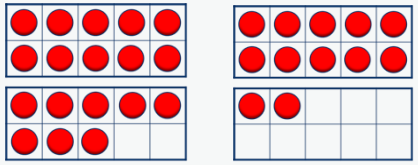
Year 2	• Recall and use addition facts to 20 fluently, and derive and use related facts up to 100
	• Add numbers using concrete objects, pictorial representations, and mentally, including:
	  • a two-digit number and 1s
	  • a two-digit number and 10s
	  • 2 two-digit numbers
	  • adding 3 one-digit numbers
	• Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.		

Progression of skills	**Key representations**		
Add ones to any number (related facts) Make links to known facts.	I know that … and …= … so … and … = … 	… more than … is … so … more than … is … 	What do you notice? Can you continue the pattern? 5 + 2 = 7 15 + 2 = 17 25 + 2 = 27…
Add three 1-digit numbers Prompt children to understand that addition can be done in any order and to make links to known facts.	… and … are a bond to 10 10 + … = … 	Double … + … = … 	What do you notice? Which addition is the easiest to calculate? 8 + 9 + 1 = 8 + 1 + 9 = 9 + 1 + 8 =

Addition

Progression of skills	Key representations		
Add across a 10 Partition the number being added to make a full ten.	… can be partitioned into … and … 	I add … to get to … then I add … 8 + 5 = 13 28 + 5 = 33 	
Add multiples of 10 Make links to known facts within ten.	… ones + … ones = … ones so … tens + … tens = … tens  3 + 2 = 5 30 + 20 = 50	What is the same? What is different? 	
Add 10s to any number Make links to known facts.	… tens + … tens = … tens … tens and … ones= … 	To add … I need to add 10 … times. 	I know that … and …= … so … and …= … 30 + 20 = 50 34 + 20 = 54

Addition

Progression of skills	Key representations
Add 2-digit numbers (not across a ten) Lining up ones and tens in columns will support with later written methods.	… ones + … ones = … ones … tens + … tens = … tens   3 ones + 1 one = 4 ones 4 tens + 2 tens = 6 tens 6 tens + 4 ones = 64 
Add 2-digit numbers (across a ten) Begin to exchange 10 ones for 1 ten.	There are …. ones, so I do/do not need to make an exchange. … ones = … ten and … ones    5 ones + 7 ones = 12 ones 12 ones = 1 ten and 2 ones 4 tens + 3 tens + 1 ten = 8 tens 8 tens and 2 ones= 82
Missing numbers Solve missing number problems and use the inverse to check.	How many more do you need to make …?  6 + □ = 10 10 − □ = 6 If … is a whole and … is a part, then … is the other part. □ + 3 = 7 7 − 3 = □  … can be partitioned into … and … 10 + 8 = 12 + □ 

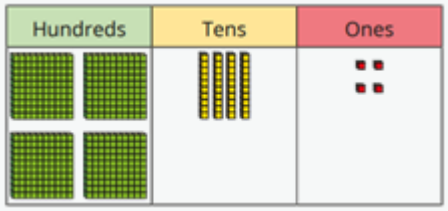
Addition

Year 3	• Add numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds. • Add numbers with up to three digits, using formal written methods of columnar addition. • Add fractions with the same denominator within 1 whole. • Calculate the time taken by particular events or tasks.
Progression of skills	**Key representations**

Add 1s, 10s or 100s to a 3-digit number

Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds column will increase by …



444 + 5 =
444 + 50 =
444 + 500 =

777 + 2 =
777 + 20 =
777 + 200 =

What patterns do you notice?

235 + 3 =
235 + 30 =
235 + 300 =

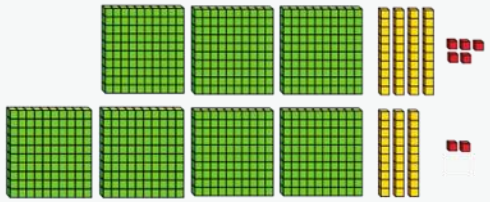
604 + 20 =
604 + 50 =
604 + 90 =

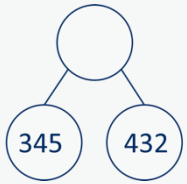
111 + ☐ = 118
111 + ☐ = 181
111 + ☐ = 811

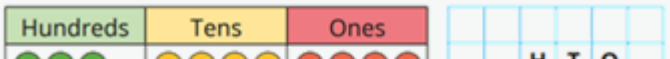
Add two numbers (no exchange)

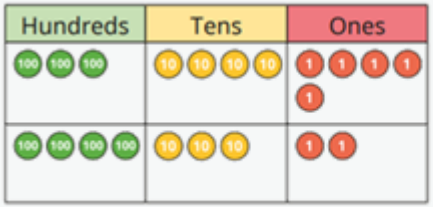
Mental strategies and introduction of formal written method.

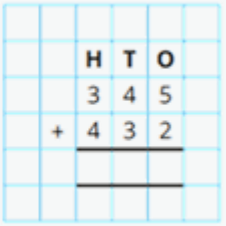
… ones + … ones = … ones
… tens + … tens = … tens
… hundreds + … hundreds = … hundreds



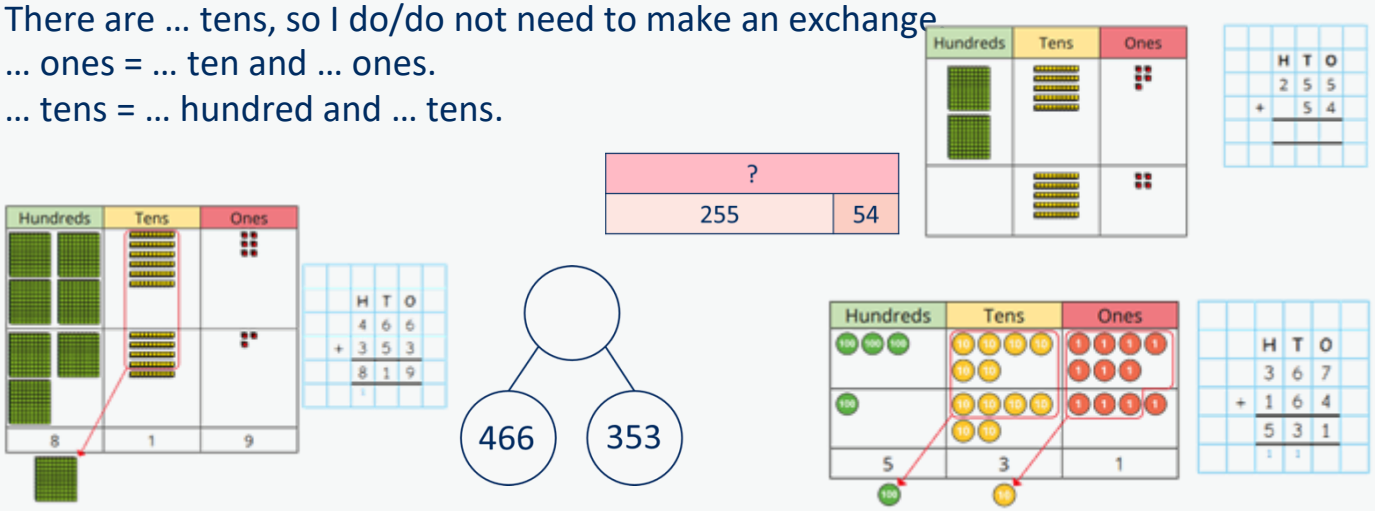
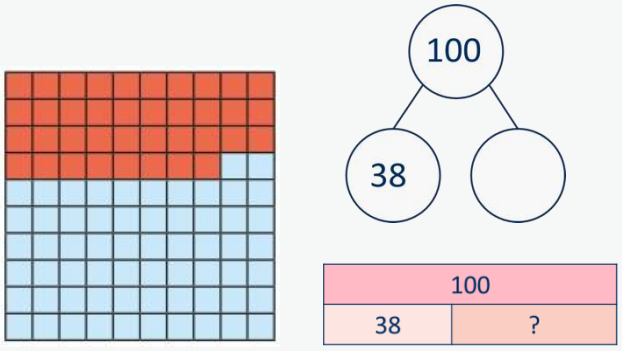
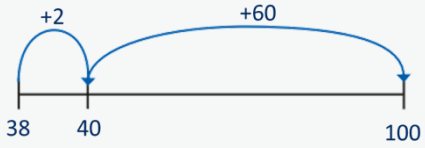




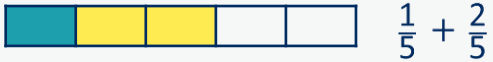
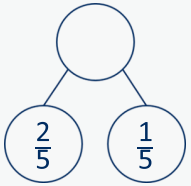
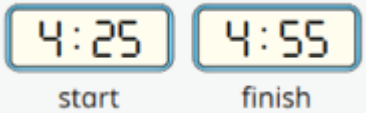
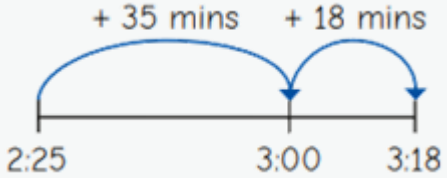




Addition

Progression of skills	Key representations
Add two numbers across a 10 or 100 Formal written method involving up to 2 exchanges including 3-digit plus 2-digit numbers.	There are … ones, so I do/do not need to make an exchange. There are … tens, so I do/do not need to make an exchange. … ones = … ten and … ones. … tens = … hundred and … tens. 
Complements to 100 Pairs of numbers which total 100	… plus … is equal to 100 
	I add … to get to the next 10, then … to get to 100  $38 + 62 = 100$ $62 + 38 = 100$ $100 = 38 + 62$ $100 = 62 + 38$

Addition

Progression of skills	Key representations
Add fractions with the same denominator within 1 whole Make links with known facts.	When adding fractions with the same denominator, I only add the numerator. … fifths + … fifths = … fifths  $\frac{1}{5} + \frac{1}{5}$  $\frac{1}{5} + \frac{2}{5}$  $\frac{1}{5} + \frac{3}{5}$  
Calculate the duration of events Find durations of time between a given start and end point. Children will need to calculate complements to 60	From … to … o'clock is … minutes. From … o'clock to … is … minutes. The total time taken is … minutes.   

Addition

Year 4	• Add numbers with up to 4 digits using a formal written method. • Solve simple measure and money problems involving fractions and decimals to 2 decimal places. • Add fractions with the same denominator.
Progression of skills	**Key representations**

Add 1s, 10s and 100s to a 4-digit number

Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds/thousands column will increase by …

$3,425 + 3 =$
$3,425 + 30 =$
$3,425 + 300 =$
$3,425 + 3,000 =$

What patterns do you notice?

$2,350 + 3 =$
$2,350 + 30 =$
$2,350 + 300 =$
$2,350 + 3,000 =$

$6,040 + 200 =$
$6,040 + 500 =$
$6,040 + 900 =$

$2,211 + \square = 2,251$
$2,211 + \square = 2,215$
$2,211 + \square = 2,511$

Add up to two 4digit numbers

Formal written method with up to 3 exchanges. Encourage children to estimate and use inverse operations to check answers to calculations.

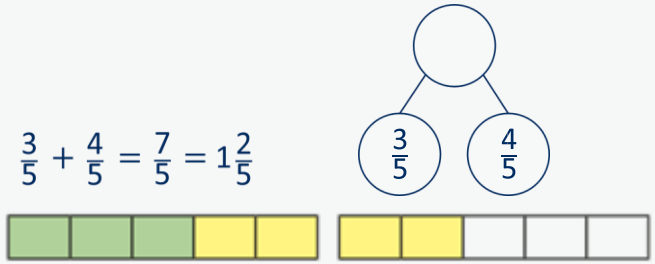
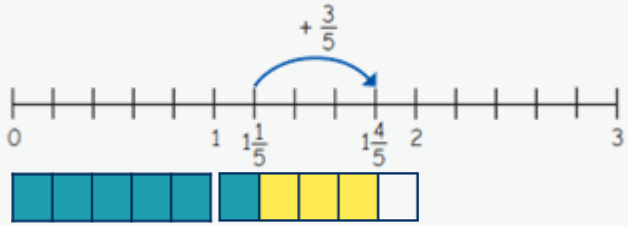
There are … ones/tens/hundreds so I do/do not need to make an exchange.

I can exchange 10 … for 1 …

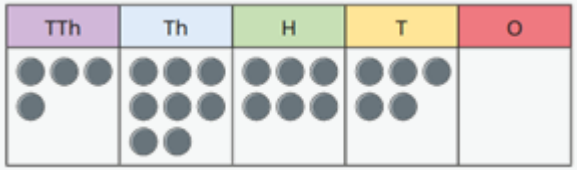
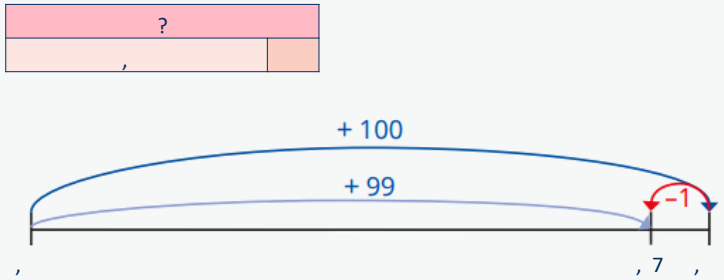
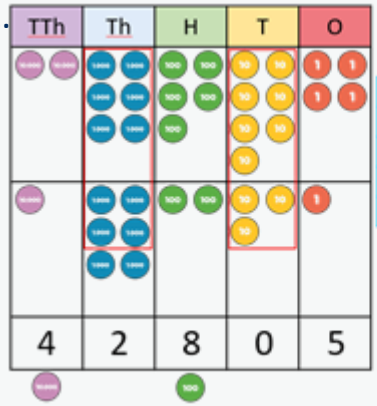
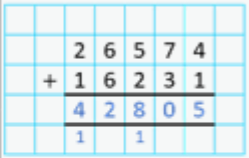
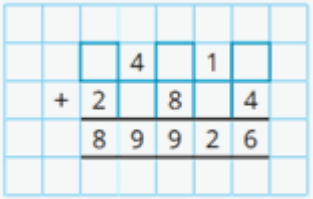
	Th	H	T	O
	4	6	7	3
+	1	5	1	8
	6	1	9	1

Addition

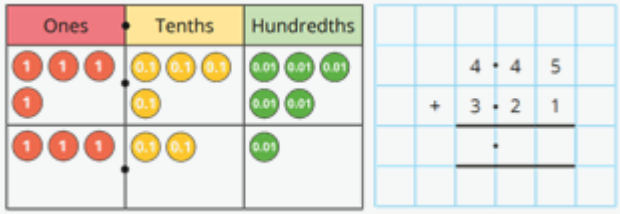
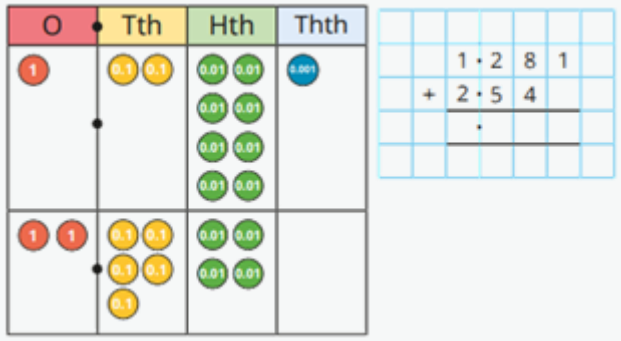
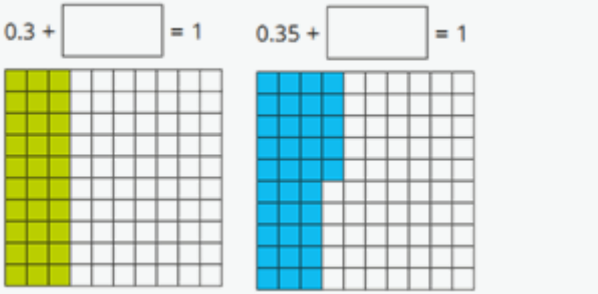
<table>
<tr><th>Progression of skills</th><th colspan="2">Key representations</th></tr>
<tr><td>Add decimal numbers in the context of money

Emphasis on partitioning and use of number lines rather than formal written calculations.</td><td>… pence + … pence = … pence
… pounds + … pounds = … pounds
 
45p + 25p = 70p
£2 + £3 = £5
£5 + 70p = £5.70</td><td>£3.25 can be partitioned into £3 + 20p + 5p
  </td></tr>
<tr><td>Add fractions and mixed numbers with the same denominator beyond 1 whole</td><td colspan="2">When adding fractions with the same denominator, I only add the numerator.
… fifths + … fifths = … fifths
$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$
 
  </td></tr>
</table>

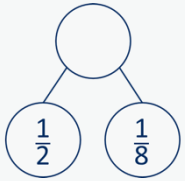
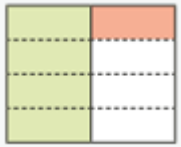
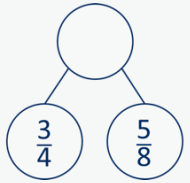
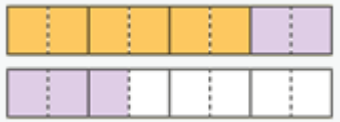
Addition

Year 5	• Add whole numbers with more than 4 digits, including using formal written methods • Add numbers mentally with increasingly large numbers. • Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 • Add fractions with the same denominator, and denominators that are multiples of the same number.
Progression of skills	**Key representations**
Add using mental strategies Add 1s, 10s, 100s, etc. to any number. Use number bonds and related facts.	 48,650 + 300 = 48,650 + 30,000 = 48,650 + 30 =
	To add …, I can add … then subtract … 
Add whole numbers with more than 4 digits Encourage children to estimate and use inverse operations to check answers to calculations.	I can exchange 10 … for 1 …   

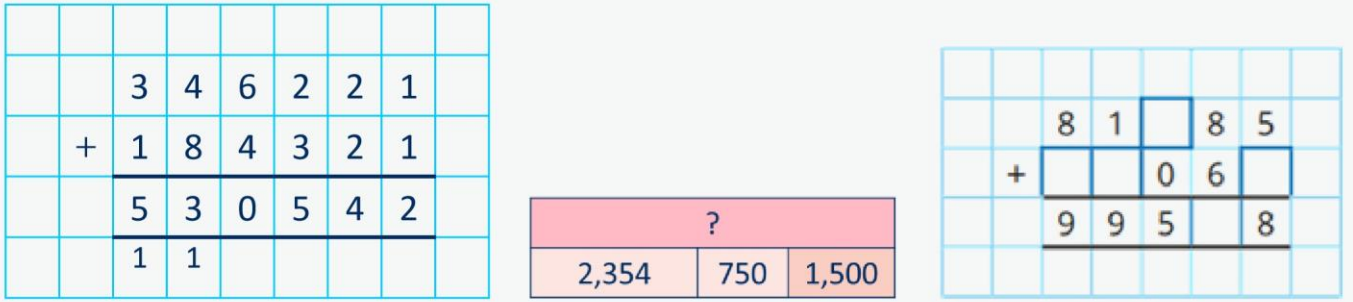
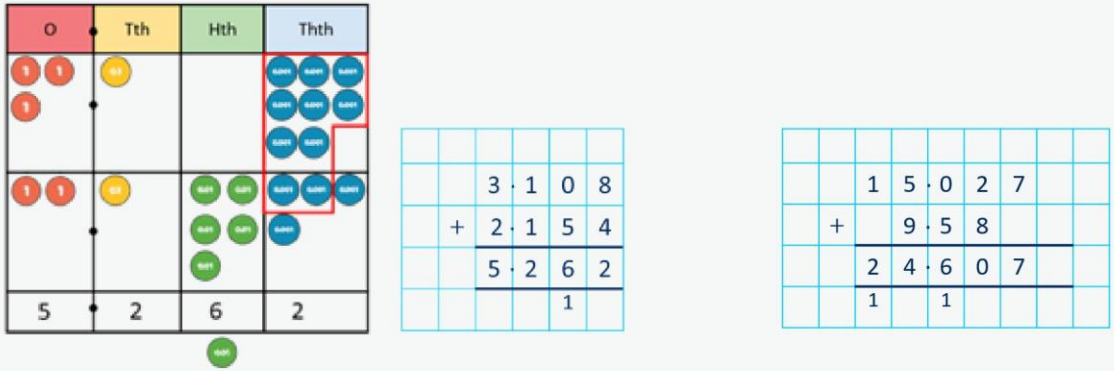
Addition

Progression of skills	Key representations
Add decimals with up to 2 decimal places Progress from the same number of decimal places to a different number of decimal places, and from no exchange to exchange.	I do/do not need to make an exchange because … I can exchange 10 … for 1 …  
Complements to 1 Pairs of numbers with up to 3 decimal places which total 1 Encourage children to make links with bonds to 10 and complements to 100 and 1,000	   $4 + 6 = 10$ $0.4 + 0.6 = 1$ $44 + 56 = 100$ $0.44 + 0.56 = 1$ $444 + 556 = 1{,}000$ $0.444 + 0.556 = 1$

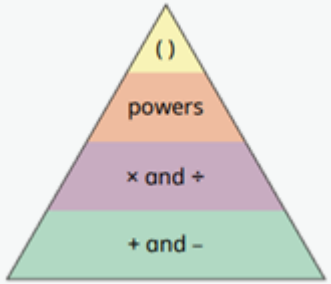
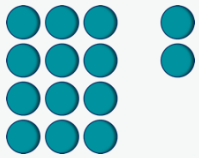
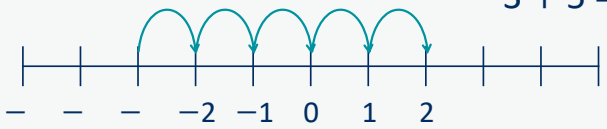
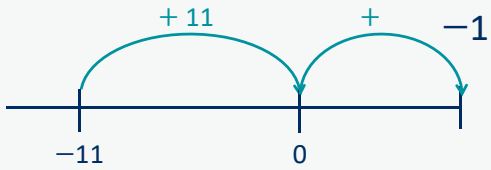
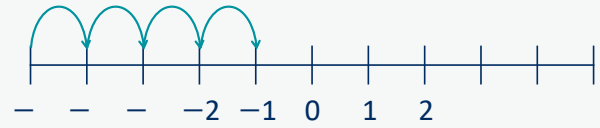
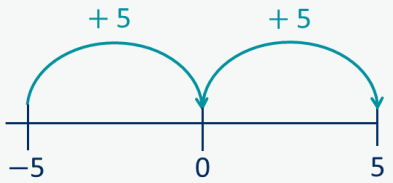
Addition

Progression of skills	Key representations
Add fractions with denominators that are a multiple of one another Encourage children to convert fractions to the same denominator before adding. Progress from adding fractions within 1 whole to adding fractions beyond 1 whole.	The denominator has been multiplied by … , so the numerator needs to be multiplied by… for the fractions to be equivalent.   $\frac{1}{2} + \frac{1}{8} = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$  $\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$   $\frac{3}{4} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8} = \frac{11}{8} = 1\frac{3}{8}$

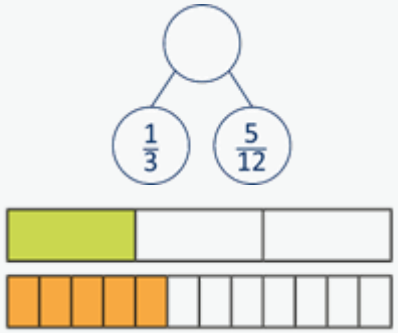
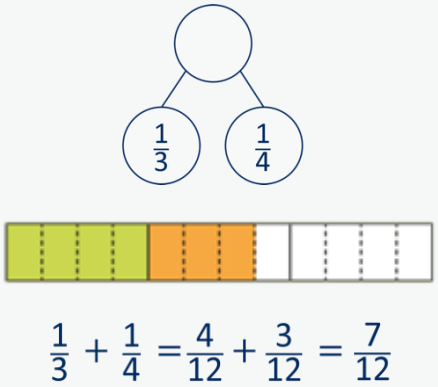
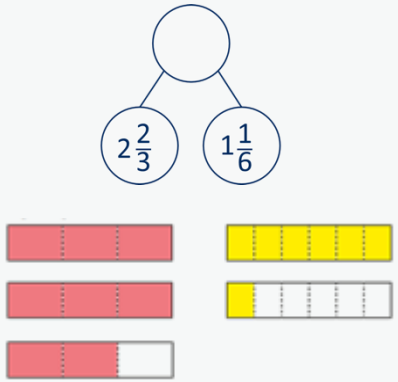
Addition

Year 6	• Add larger numbers, using the formal written method of columnar addition. • Use their knowledge of the order of operations to carry out calculations involving the 4 operations. • Calculate intervals across zero. • Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions.
Progression of skills	**Key representations**
Add integers up to 10 million Encourage children to estimate and use inverse operations to check answers to calculations.	
Add decimals with up to 3 decimal places Progress to numbers with digits in different place value columns. Encourage children to check that they have lined up the columns correctly.	I do/do not need to make an exchange because … 

Addition

Progression of skills	Key representations
Order of operations Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction. *When no brackets are shown and the operations have the same priority, work left to right.	… has greater priority than …, so the first part of the calculation I need to do is …   $(3+4)\times 2=14$  $3+4\times 2=11$  $3\times 4+2=14$
Negative numbers Children add to negative numbers and carry out calculations which cross 0	… plus … is equal to … $-3+5=2$   $-11+16=5$	 The difference between –5 and –1 is 4  The difference between –5 and 5 is 10

Addition

Progression of skills	Key representations		
Add fractions Convert fractions to the same denominator before adding. Progress from fractions where one denominator is a multiple of the other, to any fractions and then to mixed numbers.	The denominator has been multiplied by … , so the numerator needs to be multiplied by …  	The lowest common multiple of … and … is …  	…is made up of … wholes and … 

Progression of skills - Subtraction

Year group	Skill
Reception	• Conceptually subitise to 5 • 1 less • Notice the composition of numbers within 10 • Partition • Take away
Year 1	• Find a part • Take away • Bonds within 10 • Related facts within 20 • Missing numbers

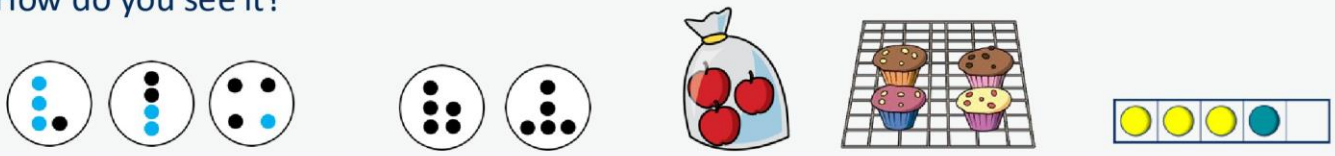
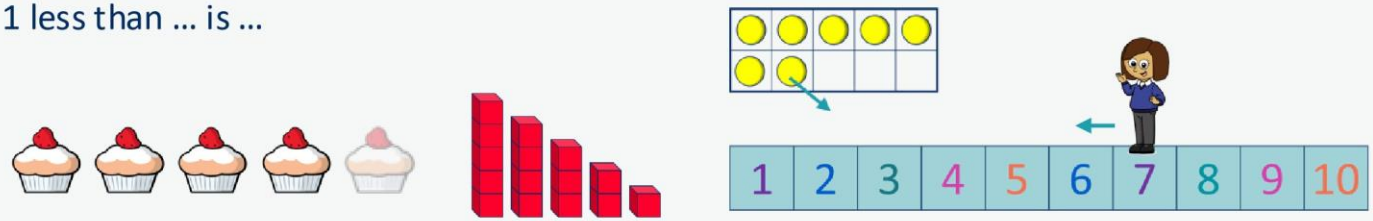
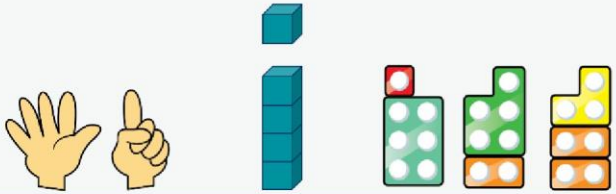
Progression of skills - Subtraction

Year group	Skill
Year 2	• Subtract 1s from any number (related facts) • Subtract across a 10 • Subtract multiples of 10 • Subtract 10s from any number • Subtract two 2-digit numbers (not across a ten) • Subtract two 2-digit numbers (across a ten) • Missing numbers
Year 3	• Subtract 1s, 10s and 100s from a 3-digit number • Subtract two numbers (no exchange) • Subtract two numbers across a 10 or 100 • Complements to 100 • Subtract fractions with the same denominator within 1 whole

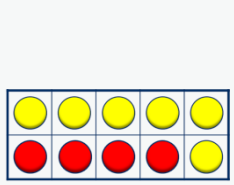
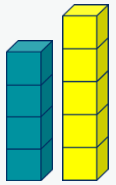
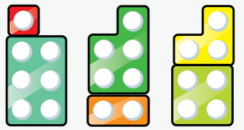
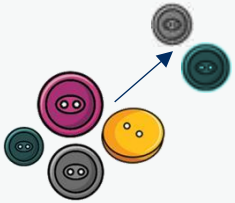
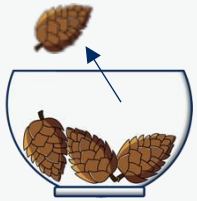
Progression of skills - Subtraction

Year group	Skill
Year 4	• Subtract 1s, 10s, 100s and 1,000s from a 4-digit number • Subtract up to two 4-digit numbers • Subtract decimal numbers in the context of money • Subtract fractions and mixed numbers with the same denominator
Year 5	• Subtract whole numbers with more than 4 digits • Subtract using mental strategies • Subtract decimals with up to 2 decimal places • Complements to 1 • Subtract fractions with denominators that are a multiple of one another
Year 6	• Subtract integers up to 10 million • Subtract decimals with up to 3 decimal places • Order of operations • Negative numbers • Subtract fractions

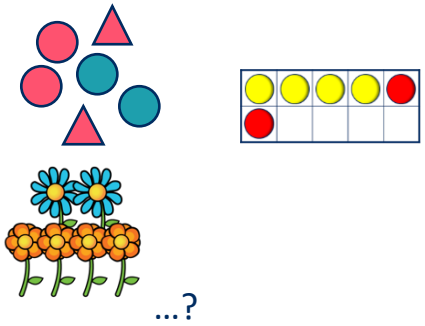
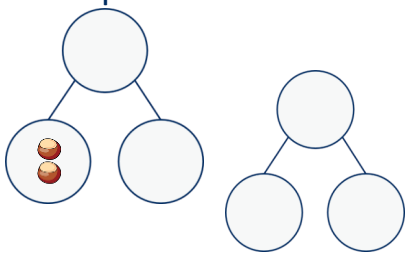
Subtraction

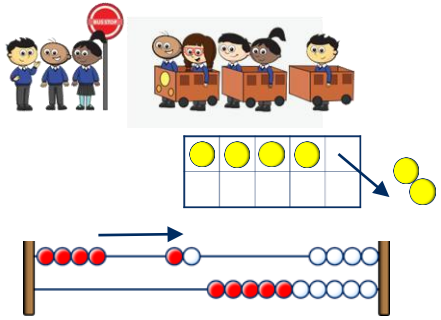
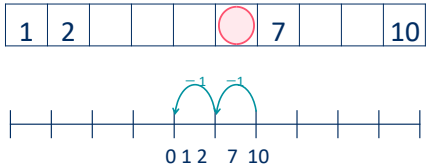
Reception	• Have a deep understanding of number to 10, including the composition of each number. • Subitise (recognise quantities without counting) up to 5 • Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (and some subtraction facts) and some number bonds to 10, including double facts.
Progression of skills	**Key representations**
Conceptually subitise to 5 Notice the parts that make up the whole.	What do you see? How do you see it? 
1 less Continue to link to stories, songs and rhymes.	1 less than … is … 
Notice the composition of numbers within 10 Link to stories, songs and rhymes.	How many…? How many…? How many altogether?  How many ways can you make…? 

Subtraction

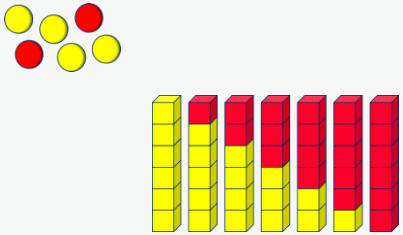
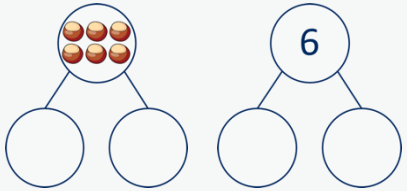
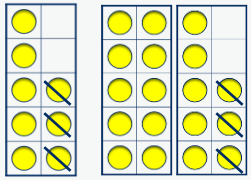
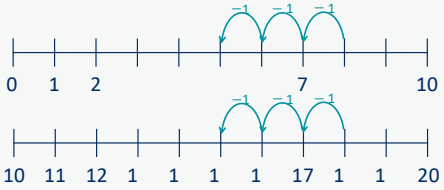
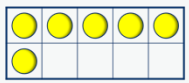
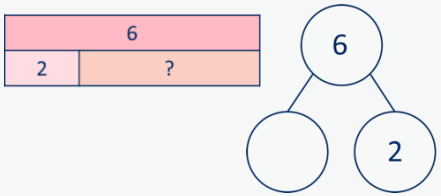
Progression of skills	Key representations	
Partition Using objects, explore different ways to partition a number into 2 or more parts.	There are … altogether. I can see … here and … there.   	… and … make …   
Take away A quantity is reduced.	First… Then… Now…    	I have … I take … away Now I have …   

Subtraction

Year 1	• Read, write and interpret mathematical statements involving subtraction (–) and equals (=) signs. • Represent and use number bonds and related subtraction facts within 20 • Subtract one-digit and two-digit numbers to 20, including zero. • Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$		
Progression of skills	**Key representations**		
Find a part Link to number bonds and known facts. E.g. $2 + 4 = 6$ so if 6 is the whole and 4 is a part, the other part must be 2	There are … in total. … are … How many are **not** …? 	… is the whole. … is a part. … is a part. 	… subtract … is equal to … … is equal to … − … $6 - 2 = 4$ $6 - 4 = 2$ $4 = 6 - 2$ $2 = 6 - 4$

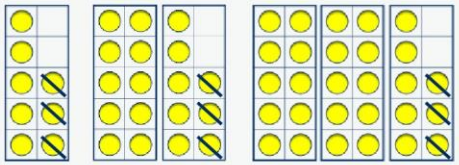
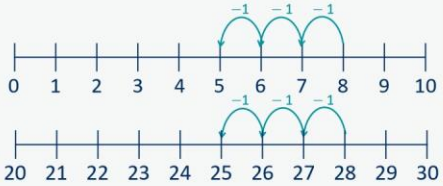
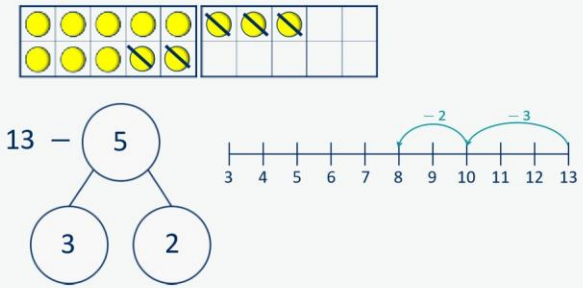
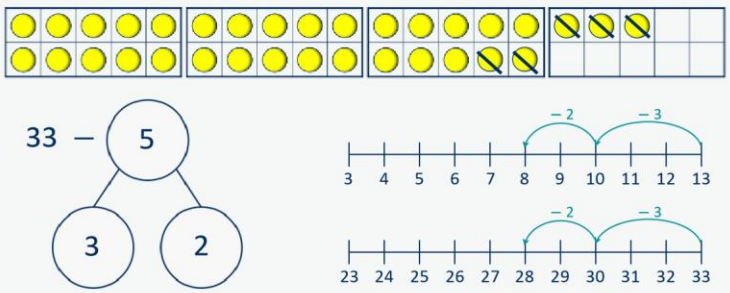
Take away A quantity is decreased.	 First… Then… Now…	I start at … I jump back … I land on … 	… minus … is equal to … … is equal to … − … $6 - 2 = 4$ $6 - 4 = 2$ $4 = 6 - 2$ $2 = 6 - 4$

Subtraction

Progression of skills	Key representations		
Bonds within 10 Focus on subtraction facts. Encourage children to notice patterns.	… is made of … and … … and … make … 	… can be partitioned into … and … 	… minus … is equal to … $6 - 0 = 6$ $6 - 1 = 5$ $6 - 2 = 4$ $6 - 3 = 3$ $6 - 4 = 2$ $6 - 5 = 1$ $6 - 6 = 0$
Related facts within 20 Make links to known facts.	I know that … minus …= … so … minus …= … 	… less than … is … so … less than … is … 	What patterns do you notice? $8 - 3 = 5$ $18 - 3 = 15$ $5 = 8 - 3$ $15 = 18 - 3$
Missing numbers Make links to known facts.	How many do you need to subtract to make …?   	If … is the whole and … is a part, the other part must be… 	… minus … is equal to … $6 - \square = 2$ $2 = 6 - \square$ 

Subtraction

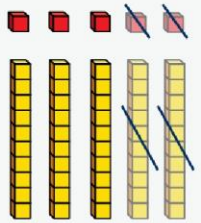
	• Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100
	• Subtract numbers using concrete objects, pictorial representations, and mentally, including:
	  • a two-digit number and 1s
	  • a two-digit number and 10s
	  • 2 two-digit numbers
	• Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.		

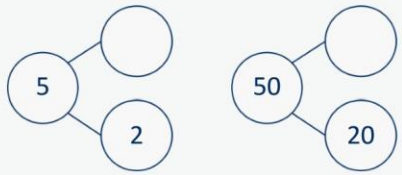
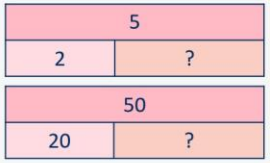
Progression of skills	**Key representations**		
Subtract ones from any number (related facts) Make links to known facts.	I know that … minus … = … so … minus … = … 	… less than … is … so … less than … is … 	What do you notice? Can you continue the pattern? $8 - 3 = 5$ $18 - 3 = 15$ $28 - 3 = 25$…
Subtract across a 10 Partition the number being subtracted to bridge through a ten.	… can be partitioned into … and … 	Make links with related facts. 	

Subtraction

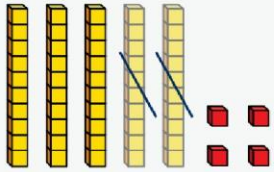
<table>
<tr><th>Progression of skills</th><th colspan="3">Key representations</th></tr>
<tr><td>Subtract multiples of 10

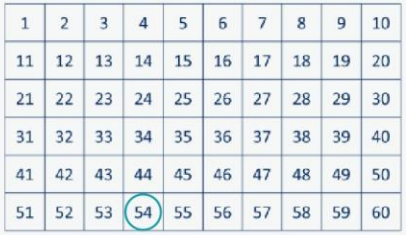
Make links to known facts within ten.</td><td>... ones − ... ones = ... ones
so ... tens − ... tens = ... tens

 
5 − 2 = 3
50 − 20 = 30</td><td colspan="2">What is the same?
What is different?

 
 
 
  </td></tr>
<tr><td>Subtract 10s from any number

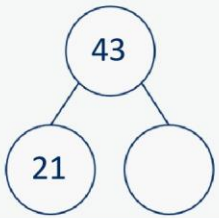
Make links to known facts.</td><td>... tens − ... tens = ... tens
... tens and ... ones = ...

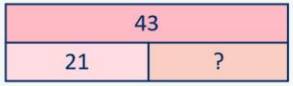
  </td><td>To subtract ... I need to subtract 10 ... times.

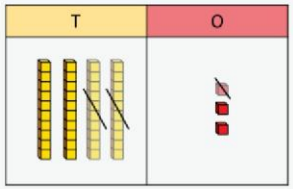
  </td><td>I know that ... minus ... = ...
so ... minus ... = ...

50 − 20 = 30
54 − 20 = 34</td></tr>
</table>

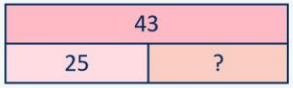
Subtraction

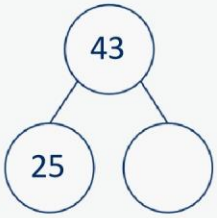
<table>
<tr><th>Progression of skills</th><th>Key representations</th></tr>
<tr><td>Subtract two 2-digit numbers
(not across a ten)</td><td>
... ones − ... ones = ... ones
... tens − ... tens = ... tens


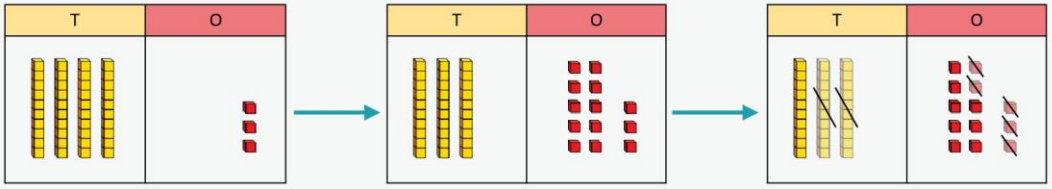




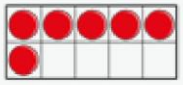
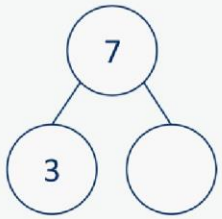
3 ones − 1 one = 2 ones
4 tens − 2 tens = 2 tens
2 tens and 2 ones = 22
</td></tr>
<tr><td>Subtract two 2-digit numbers
(across a ten)

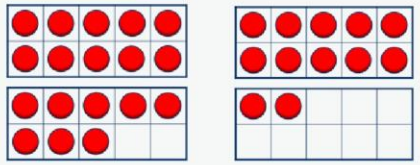
Begin to exchange 1 ten for 10 ones.</td><td>
I need to make an exchange because I do not have enough ones to subtract ... ones.






3 ones − 5 ones
(I need to exchange 1 ten for 10 ones)
13 ones − 5 ones = 8 ones
3 tens − 2 tens = 1 ten
1 ten and 8 ones = 18
</td></tr>
<tr><td>Missing numbers

Solve missing number problems and use the inverse to check.</td><td>
How many do you need to subtract to make ...?

10 − □ = 6
6 + □ = 10
</td><td>
If ... is a whole and ... is a part, then ... is the other part.
7 − 3 = □
□ + 3 = 7


</td><td>
... can be partitioned into ... and ...
18 − □ = 12 + 2

</td></tr>
</table>

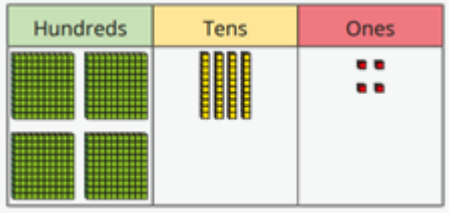
Subtraction

Year 3	• Subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds. • Subtract numbers with up to three digits, using formal written methods. • Subtract fractions with the same denominator within 1 whole.
Progression of skills	**Key representations**

Subtract 1s, 10s and 100s from a 3-digit number

Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds column will decrease by …



444 – 2 =
444 – 20 =
444 – 200 =

777 – 4 =
777 – 40 =
777 – 400 =

What patterns do you notice?

235 – 3 =
235 – 30 =
235 – 300 =

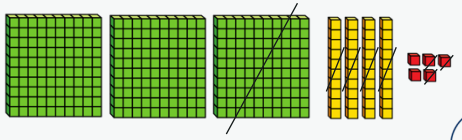
624 – 20 =
654 – 50 =
694 – 90 =

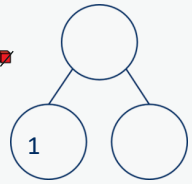
118 – ☐ = 111
181 – ☐ = 111
811 – ☐ = 111

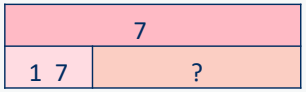
Subtract two numbers (no exchange)

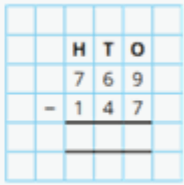
Mental strategies and introduction of formal written method.

… ones – … ones = … ones
… tens – … tens = … tens
… hundreds – … hundreds = … hundreds

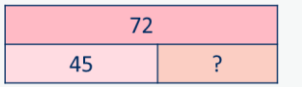
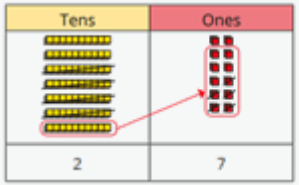
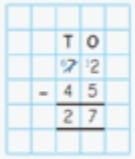
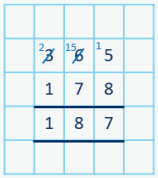
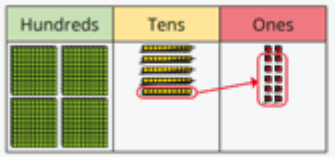
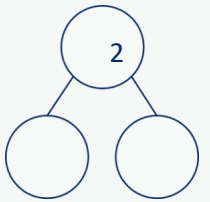
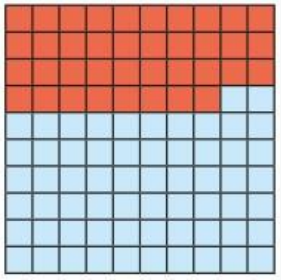
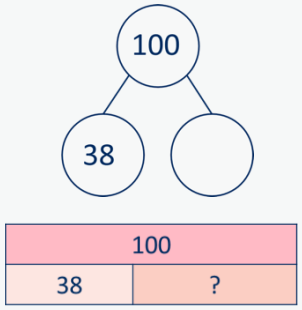
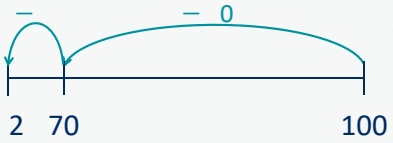




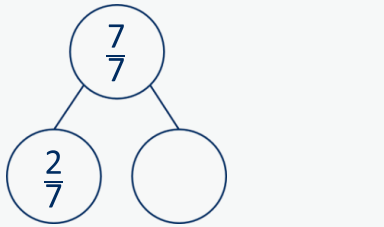




Subtraction

Progression of skills	Key representations	
Subtract two numbers across a 10 or 100 Formal written method involving up to 2 exchanges including 3-digit subtract 2-digit numbers.	I need to subtract … ones. I do/do not need to make an exchange. I need to subtract … tens. I do/do not need to make an exchange. I can exchange 1 … for 10 …         	
Complements to 100 Focus on subtraction facts. Encourage children to notice patterns.	100 minus … is equal to …   	I subtract … tens, then I subtract … ones.  $100 - 38 = 62$ $100 - 62 = 38$ $62 = 100 - 38$ $38 = 100 - 62$

Subtraction

Progression of skills	Key representations
Subtract fractions with the same denominator within 1 whole Make links with known facts.	When subtracting fractions with the same denominator, I only subtract the numerator. … fifths − … fifths = … fifths  $\frac{5}{5} - \frac{1}{5}$  $\frac{4}{5} - \frac{1}{5}$  $\frac{3}{5} - \frac{1}{5}$  

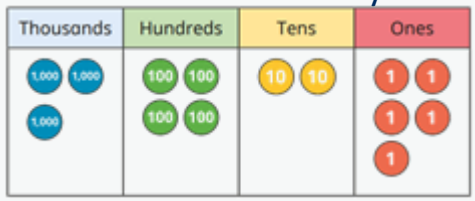
Subtraction

Year 4	• Subtract numbers with up to 4 digits using a formal written method. • Solve simple measure and money problems involving fractions and decimals to 2 decimal places. • Subtract fractions with the same denominator.
Progression of skills	**Key representations**

Subtract 1s, 10s, 100s and 1,000s from a4-digit number

Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds/thousands column will decrease by …



$3,425 - 2 =$ $3,425 - 200 =$
$3,425 - 20 =$ $3,425 - 2,000 =$

What patterns do you notice?

$4,356 - 3 =$
$4,356 - 30 =$
$4,356 - 300 =$
$4,356 - 3,000 =$

$6,940 - 200 =$
$6,940 - 300 =$
$6,940 - 400 =$

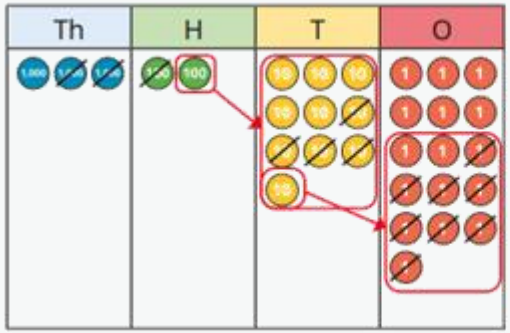
$4,433 - \square = 4,430$
$4,433 - \square = 4,033$
$4,433 - \square = 4,403$

Subtract up to two 4digit numbers

Formal written method with up to 3 exchanges. Encourage children to estimate and use inverse operations to check answers to calculations.

I need to subtract… ones/tens/hundreds. I do/do not need to make an exchange.

I can exchange 1… for 10…

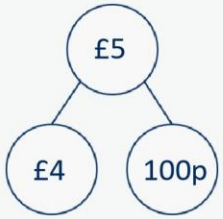
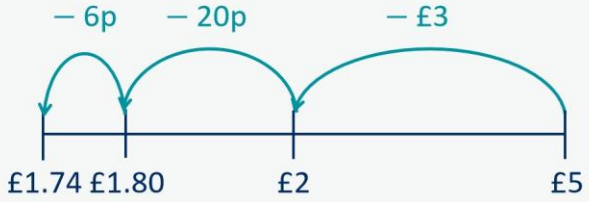


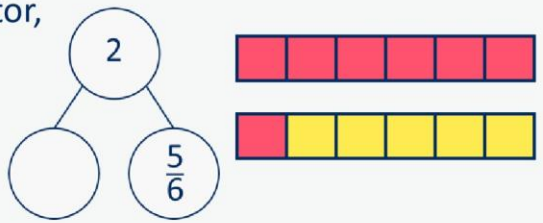
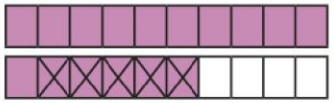
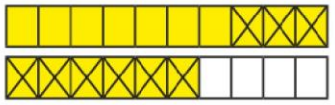
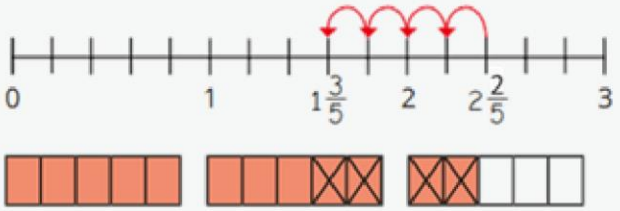
	Th	H	T	O
	3	12	9	16
-	2	1	4	8
	1	0	5	8

Subtraction

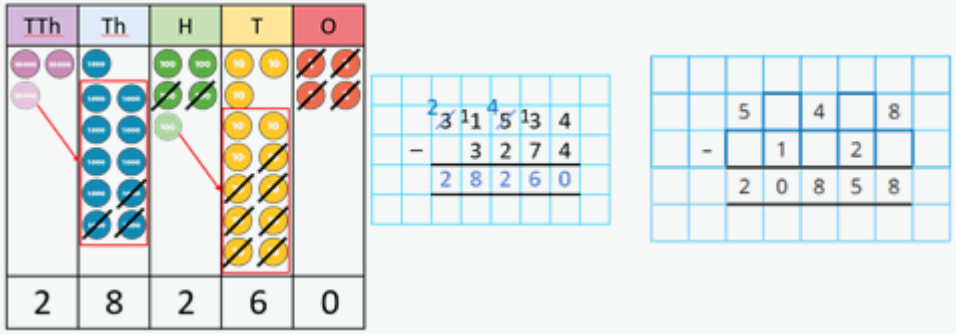
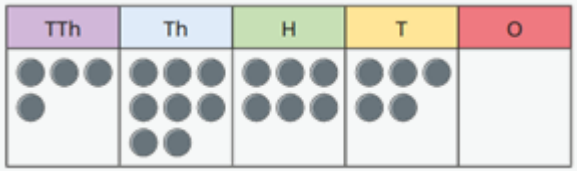
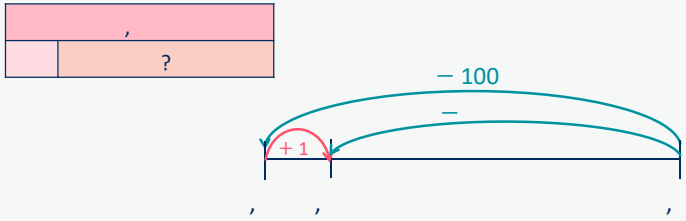
<table>
<tr><th>Progression of skills</th><th colspan="2">Key representations</th></tr>
<tr><td>Subtract decimal numbers in the context of money

Emphasis here is on partitioning and use of number lines rather than formal written calculations.</td><td>I can partition £... into £... and 100p
£... − £... = £...
100p − ...p = ...p

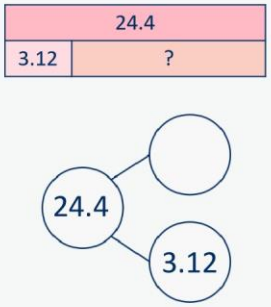
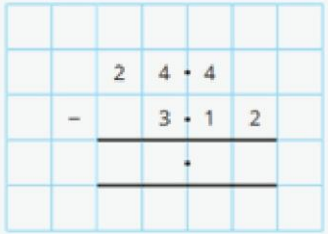
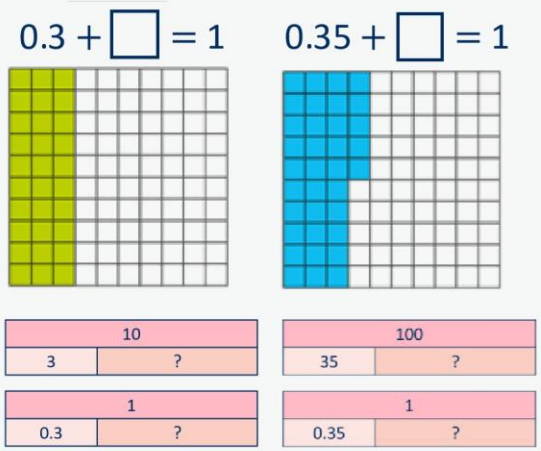
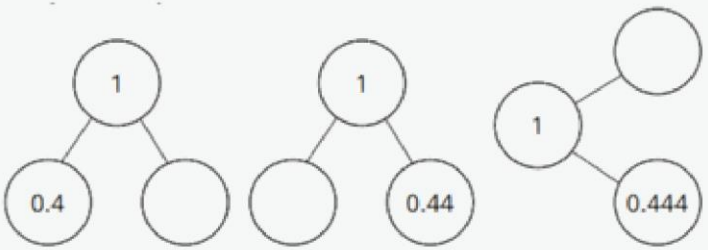
£5 − £3.26
£4 − £3 = £1
100p − 26p = 74p
£5 - £3.26 = £1.74
  </td><td>£3.26 can be partitioned into £3 + 20p + 6p
  </td></tr>
<tr><td>Subtract fractions and mixed numbers with the same denominator

Include subtracting fractions from wholes.</td><td colspan="2">When subtracting fractions with the same denominator, I only subtract the numerator.
... tenths − ... tenths = ... tenths
 
  $\frac{16}{10} - \frac{5}{10}$
  $\frac{16}{10} - \frac{9}{10}$
  </td></tr>
</table>

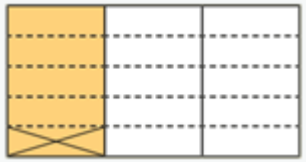
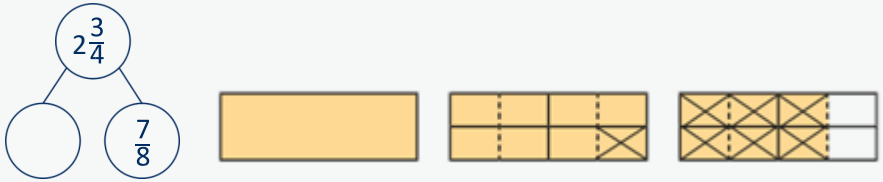
Subtraction

Year 5	• Subtract whole numbers with more than 4 digits. • Subtract numbers mentally with increasingly large numbers. • Subtract decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1 • Subtract fractions with the same denominator, and denominators that are multiples of the same number.
Progression of skills	**Key representations**
Subtract whole numbers with more than 4 digits Encourage children to estimate and use inverse operations to check answers to calculations.	I can exchange 1 … for 10 … 
Subtract using mental strategies Subtract 1s, 10s, 100s etc from any number. Use number bonds and related facts.	 48,650 − 300 = 48,650 − 30,000 = 48,650 − 30 =
	To subtract …, I can subtract … then add … 

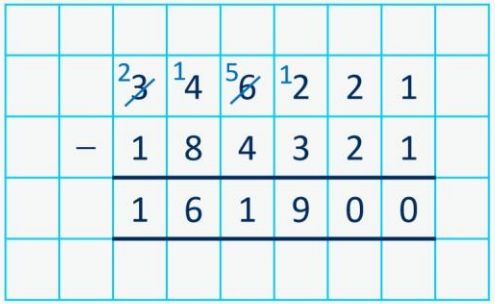
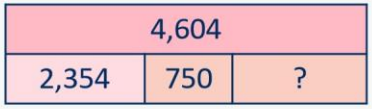
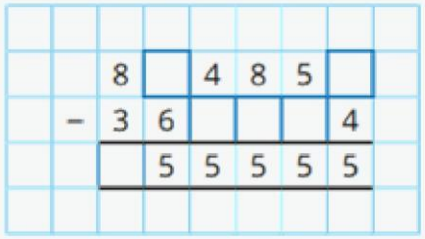
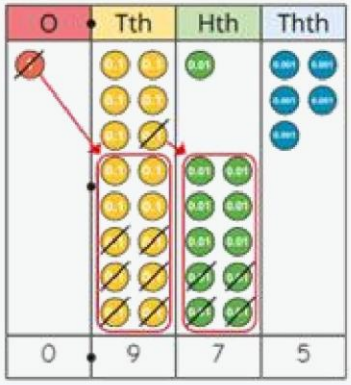
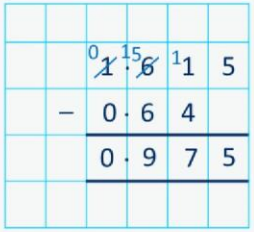
Subtraction

Progression of skills	Key representations
Subtract decimals with up to 2 decimal places Progress from the same number of decimal places to a different number of decimal places and from no exchange to exchange.	  
Complements to 1 Encourage children to make links with bonds to 10 and complements to 100 and 1,000 when finding a missing part or subtracting from 1	  $10 - 4 = 6$ $1 - 0.4 = 0.6$ $100 - 44 = 56$ $1 - 0.44 = 0.56$ $1{,}000 - 444 = 556$ $1 - 0.444 = 0.556$

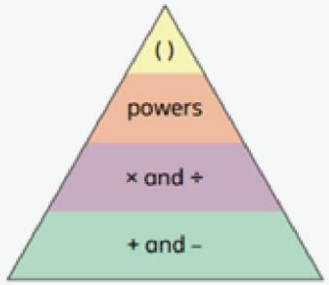
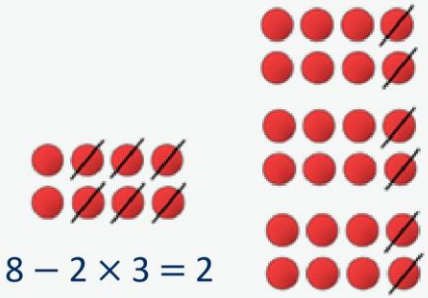
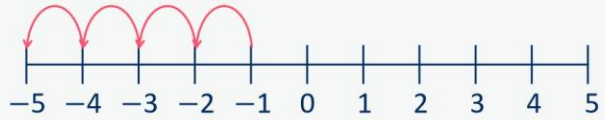
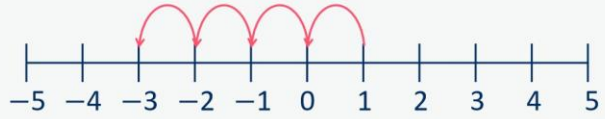
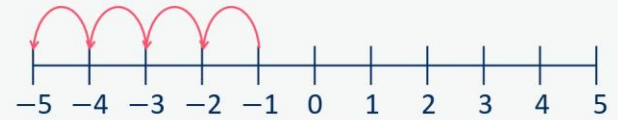
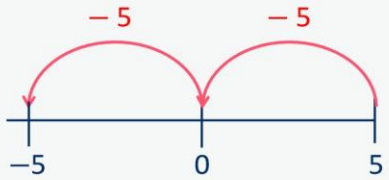
Subtraction

Progression of skills	Key representations
Subtract fractions with denominators that are a multiple of one another Convert fractions to the same denominator before subtracting. Progress from subtracting fractions within 1 whole to subtracting from a mixed number.	The denominator has been multiplied by … , so the numerator needs to be multiplied by… for the fractions to be equivalent.  $\frac{1}{3}-\frac{1}{15}=\frac{5}{15}-\frac{1}{15}=\frac{4}{15}$  $\frac{2}{3}-\frac{2}{9}=\frac{6}{9}-\frac{2}{9}=\frac{4}{9}$ 

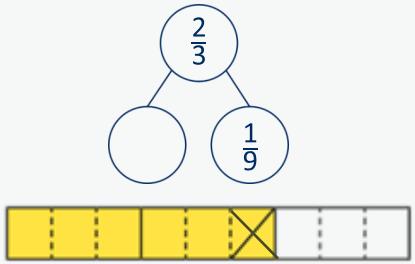
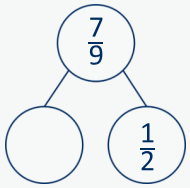
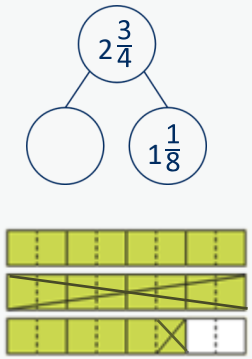
Subtraction

Year 6	• Subtract larger numbers, using the formal written methods of columnar subtraction. • Use their knowledge of the order of operations to carry out calculations involving the 4 operations. • Calculate intervals across zero. • Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.
Progression of skills	**Key representations**
Subtract integers up to 10 million Encourage children to estimate and use inverse operations to check answers to calculations.	  
Subtract decimals with up to 3 decimal places Progress from the same number of decimal and whole number places to a different number of decimal and whole number places.	I do/do not need to make an exchange because …   

Subtraction

Progression of skills	Key representations
Order of operations Children learn the order of priority for operations in a calculation. Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.	… has greater priority than … , so the first part of the calculation I need to do is …   $8 - 2 \times 3 = 2$ $(8 - 2) \times 3 = 18$  $8 - 2^2 = 4$
Negative numbers Children subtract from positive and negative numbers and calculate intervals across 0	… minus … is equal to … $-1 - 4 = -5$  $1 - 4 = -3$  	 The difference between −5 and −1 is 4  The difference between 5 and −5 is 10

Subtraction

Progression of skills	Key representations		
Subtract fractions Convert fractions to the same denominator before subtracting. Progress from fractions where one denominator is a multiple of the other, to any fractions and then subtracting from a mixed number.	The denominator has been multiplied by … , so the numerator needs to be multiplied by…  $\frac{2}{3} - \frac{1}{9} = \frac{6}{9} - \frac{1}{9} = \frac{5}{9}$	The lowest common multiple of … and … is …  $\frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}$	… is made up of … wholes and …  $2\frac{3}{4} - 1\frac{1}{8} = 1\frac{5}{8}$

Progression of skills - Multiplication

Year group	Skill
Reception	• Double to 10 • Make equal groups
Year 1	• Count in 2s, 5s and 10s • Add equal groups • Make arrays • Make doubles

Progression of skills - Multiplication

Year group	Skill
Year 2	• Link repeated addition and multiplication • Use arrays • Double • The 2 times-table • The 10 times-table • The 5 times-table • Missing numbers
Year 3	• The 3 times-table • The 4 times-table • The 8 times-table • Related facts • Multiply a 2-digit number by a 1-digit number - no exchange • Multiply a 2-digit number by a 1-digit number - with exchange • Scaling • Correspondence problems

Progression of skills - Multiplication

Year group	Skill
Year 4	• Times-table facts to 12 × 12 • Multiply by 1 and 0 • Multiply 3 numbers • Factor pairs • Multiply by 10 and 100 • Related facts • Mental strategies • Multiply a 2 or 3-digit number by a 1-digit number • Scaling • Correspondence problems

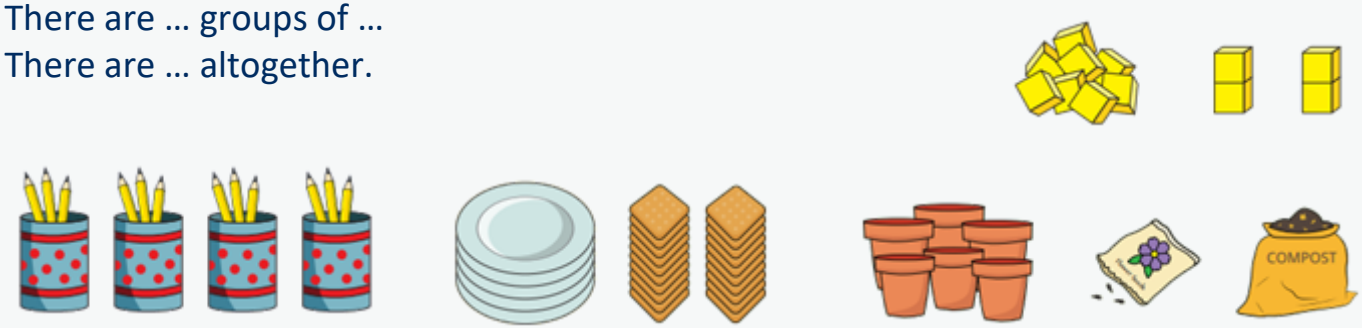
Progression of skills - Multiplication

Year group	Skill
Year 5	• Multiples and factors • Square and cube numbers • Multiply numbers up to 4 digits by a 1-digit number • Multiply numbers up to 4 digits by a 2-digit number • Multiply by 10, 100 and 1,000 • Mental strategies • Multiply fractions by a whole number • Multiply mixed numbers by a whole number • Find the whole

Progression of skills - Multiplication

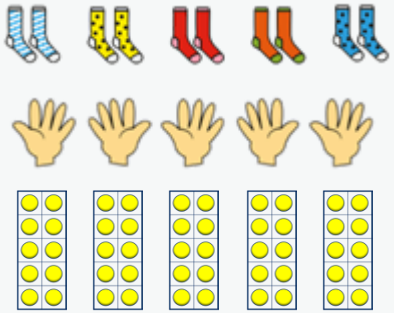
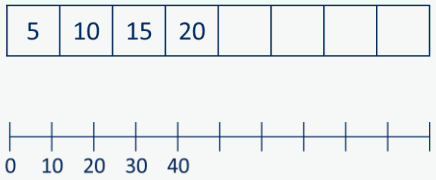
Year group	Skill
Year 6	• Multiply numbers up to 4 digits by a 2-digit number • Multiply by 10, 100 and 1,000 • Order of operations • Multiply decimals by integers • Multiply fractions by fractions • Find the whole • Calculations involving ratio

Multiplication

Reception	• Have a deep understanding of number to 10, including the composition of each number. • Subitise (recognise quantities without counting) up to 5 • Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts. • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.
Progression of skills	**Key representations**
Double to 10 Prompt children to notice that double means twice as many and to notice that there are two equal groups.	Double … is … … is double … 
Make equal groups Provide opportunities to make equal groups when tidying up or during snack time. Encourage children to check that each group has the same amount.	There are … groups of … There are … altogether. 

Multiplication

<table>
<tr><td>Year 1</td><td colspan="3">• Count in multiples of twos, fives and tens.
• Solve one-step problems involving multiplication, using concrete objects, pictorial representations and arrays with the support of the teacher.</td></tr>
<tr><td>Progression of skills</td><td colspan="3">Key representations</td></tr>
<tr><td>Count in 2s, 5s and 10s

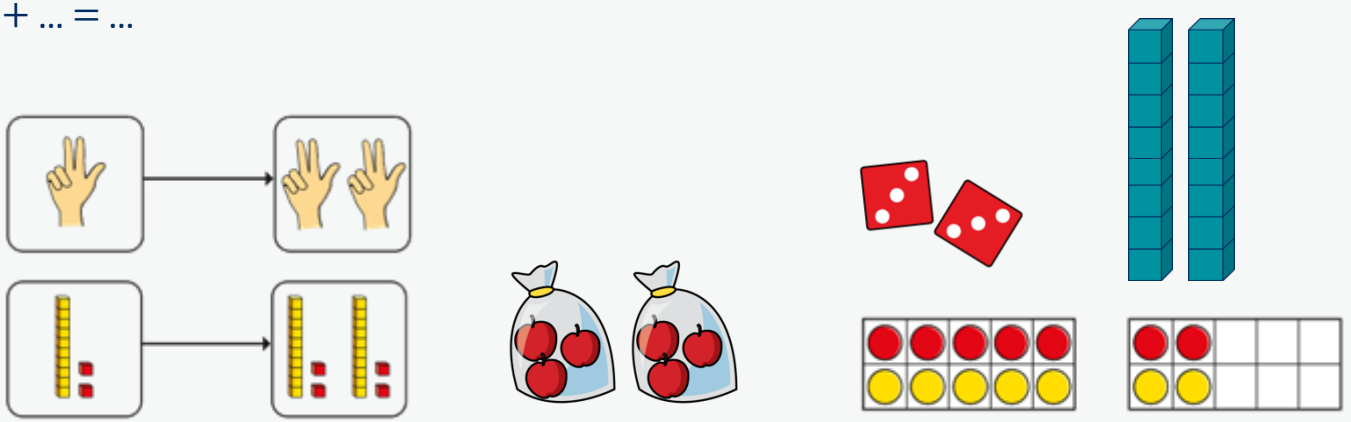
Begin by counting objects that naturally come in 2s, 5s and 10s, for example pairs of socks or fingers.</td><td>There are … equal groups of …
There are … altogether.
  </td><td>Continue to colour in …s
What do you notice?
 
<table><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr><tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr><tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr><tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr><tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr></table></td><td>Complete the number track/number line by counting in …s.
 
<table><tr><td>5</td><td>10</td><td>15</td><td>20</td><td></td><td></td><td></td><td></td></tr></table>
0 10 20 30 40</td></tr>
<tr><td>Add equal groups
(repeated addition)

Children should be able to write a repeated addition to represent equal groups and to draw pictures or use objects to represent a repeated addition.</td><td>There are … groups of …
There are … altogether.
 
10 + 10 + 10 = 30

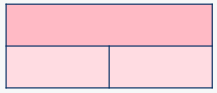
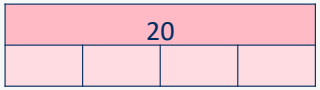
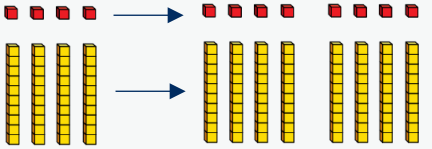
 
5 + 5 + 5 + 5 = 20</td><td colspan="2">What is the same? What is different?
2 + 2 + 2 =
5 + 5 + 5 =
10 + 10 + 10 =

Use objects or a drawing to represent the equal groups and find how many in total.</td></tr>
</table>

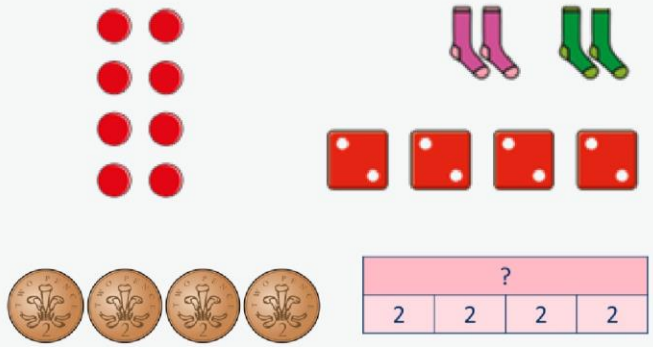
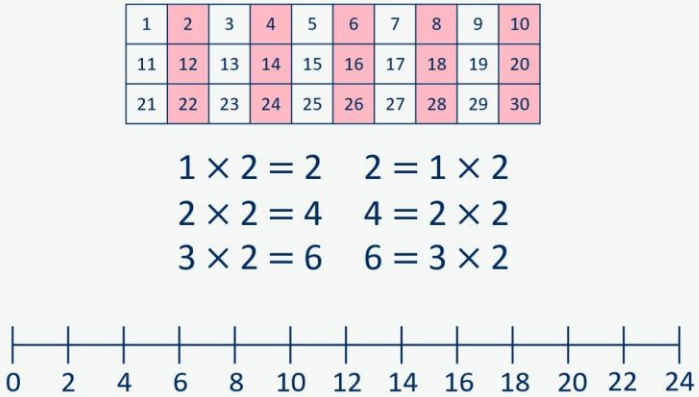
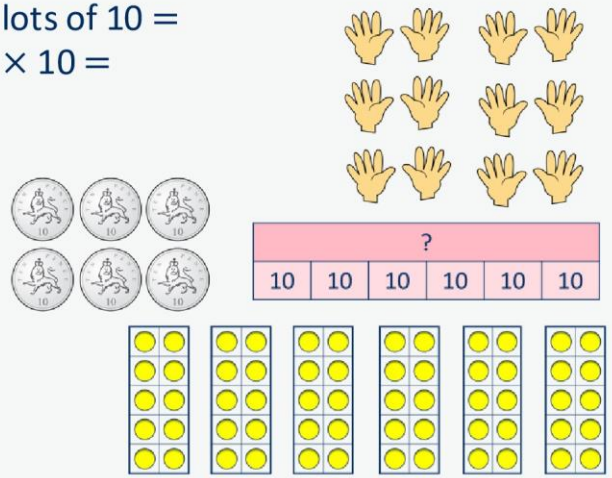
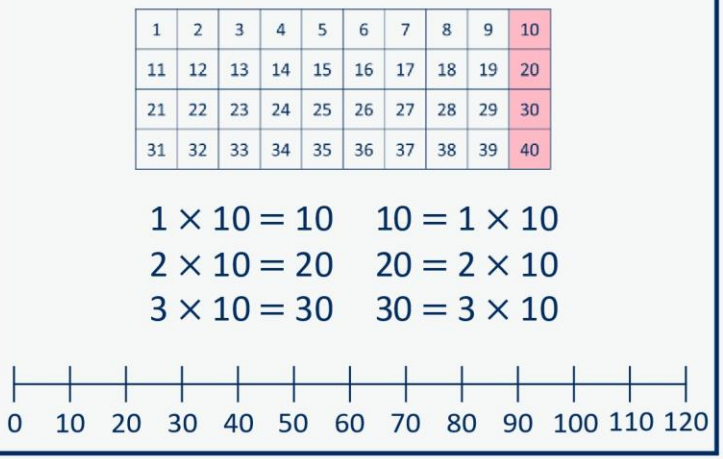
Multiplication

Progression of skills	Key representations
Make arrays Children use their knowledge of adding equal groups to arrange objects in columns and rows.	There are … rows of … There are … altogether. There are … columns of … There are … altogether. 
Make doubles Children understand that doubles are two equal groups. Children may begin to explore doubles beyond 20 using base 10	Double … is … … + … = … 

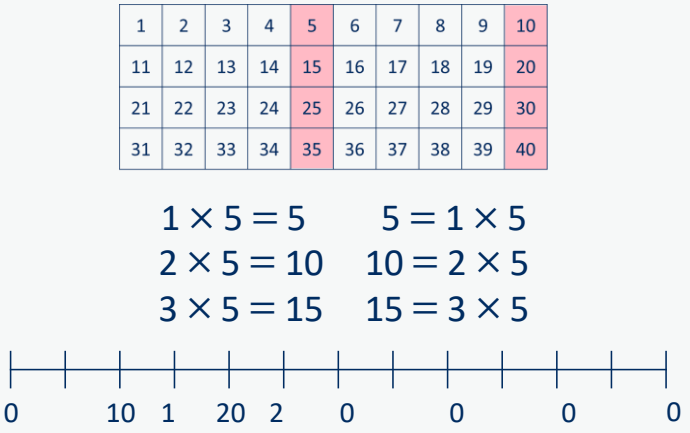
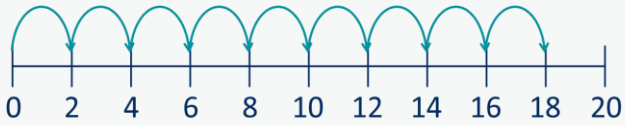
Multiplication

Year 2	• Recall and use multiplication facts for the 2, 5 and 10 multiplication tables. • Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs. • Show that multiplication of two numbers can be done in any order (commutative).
Progression of skills	**Key representations**
Link repeated addition and multiplication Encourage children to make the link between repeated addition and multiplication.	There are … equal groups with … in each group. There are … altogether.   3 + 3 = 6 2 × 3 = 6   5 + 5 + 5 + = 20 4 × 5 = 20
Use arrays Encourage children to see that multiplication is commutative.	There are … rows with … in each row. There are … columns with … in each column.  3 lots of 5 = 15 5 + 5 + 5 = 15 5 lots of 3 = 15 3 + 3 + 3 + 3 + 3 = 15
	I can see … × … and … × … 3 × 5 = 15 5 × 3 = 15 3 × 5 = 5 × 3
Double Encourage children to make links with related facts.	Double … is …  Double 4 = 4 + 4 Double 4 is 8
	Double … is … so double … is …  Double 4 is 8 Double 40 is 80

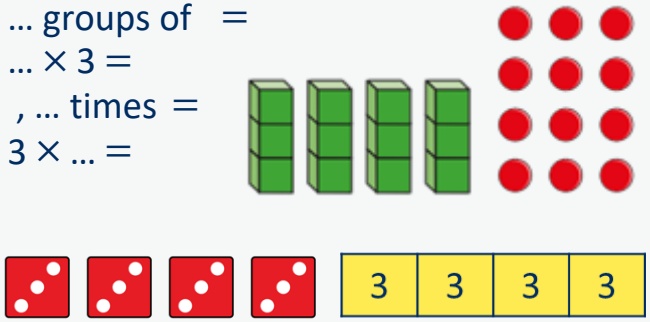
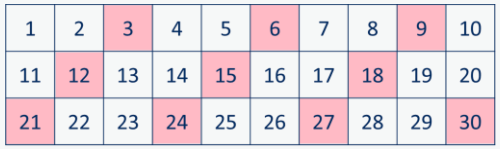
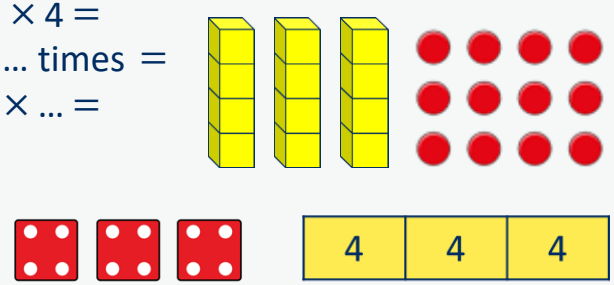
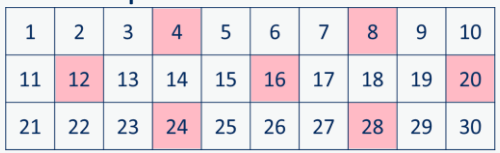
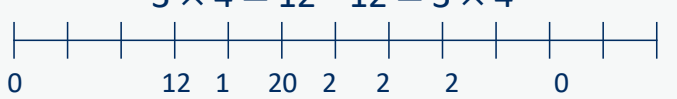
Multiplication

Progression of skills	Key representations	
The 2 times-table Encourage daily counting in multiples both forwards and back. Notice that all multiples of 2 are even numbers.	… lots of 2 = … × 2 = 	… times 2 is equal to … 
The 10 times-table Encourage daily counting in multiples both forwards and back. Notice the pattern in the numbers.	… lots of 10 = … × 10 = 	… times 10 is equal to … 

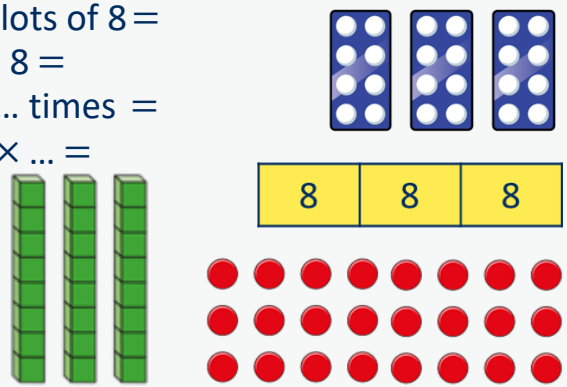
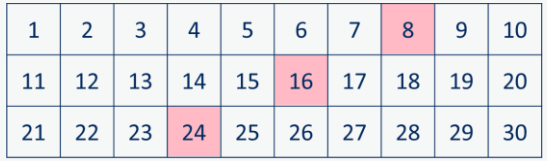
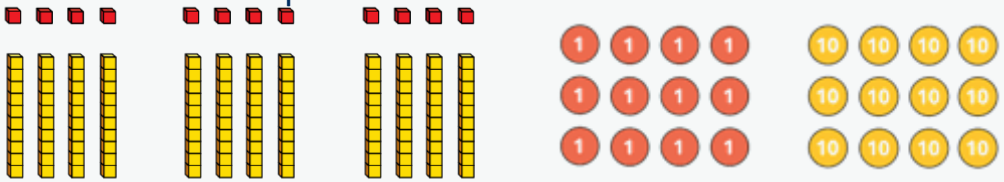
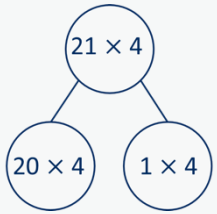
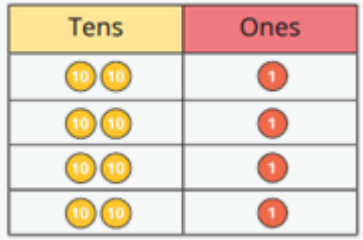
Multiplication

Progression of skills	Key representations	
The 5 times-table Encourage daily counting in multiples both forwards and back. Notice the pattern in the numbers.	… lots of … = … × 5 =  	… times … is equal to …  1 × 5 = 5 5 = 1 × 5 2 × 5 = 10 10 = 2 × 5 3 × 5 = 15 15 = 3 × 5
Missing numbers Make links to known facts.	… is equal to … groups of … 18 socks, how many pairs?   	… times … is equal to … □ × 2 = 18 18 = 2 × □

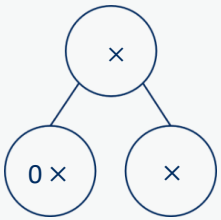
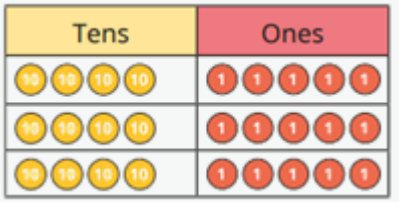
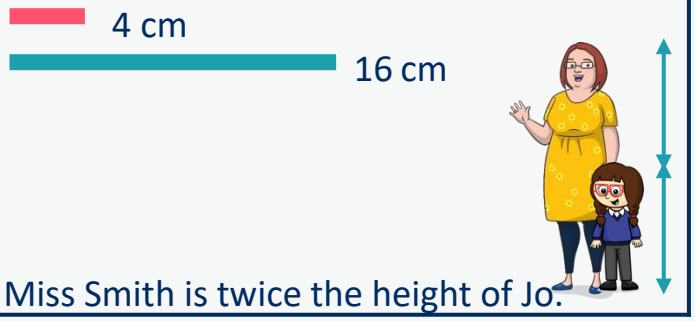
Multiplication

Year 3	• Recall and use multiplication facts for the 3, 4 and 8 multiplication tables. • Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for twodigit numbers times onedigit numbers, using mental and progressing to formal written methods. • Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
Progression of skills	**Key representations**
The 3 timestable Encourage daily counting in multiples both forwards and back.	… groups of … = … × 3 = , … times … = 3 × … =  … times … is equal to …  4 × 3 = 12 12 = 4 × 3 
The 4 timestable Encourage daily counting in multiples both forwards and back. Encourage children to notice links between the 2 and 4 timestables.	… groups of … = … × 4 = , … times … = 4 × … =  … times … is equal to …  3 × 4 = 12 12 = 3 × 4 

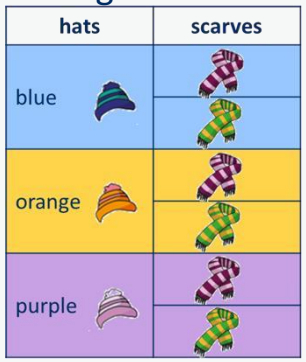
Multiplication

Progression of skills	Key representations
The 8 times-table Encourage daily counting in multiples both forwards and back. Encourage children to notice links between the 2, 4 and 8 times-tables.	… lots of 8 = × 8 = , … times = 8 × … =  … times is equal to …  3 × 8 = 24 24 = 3 × 8 
Related facts Use knowledge of multiplying by 10 to scale times-table facts.	… × … ones is equal to … ones so … × … tens is equal to … tens.  3 × 4 = 12 3 × 40 = 120
Multiply a 2-digit number by a 1-digit number- no exchange Children apply their understanding of partitioning to represent and solve calculations using the expanded method.	… tens multiplied by … is equal to … tens. …ones multiplied by … is equal to … ones.  30 × 2 = 60 2 × 2 = 4 32 × 2 = 64  

Multiplication

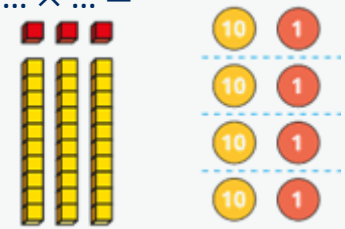
Progression of skills	Key representations	
Multiply a 2-digit number by a 1-digit number- with exchange Children apply their understanding of partitioning to represent and solve calculations using the expanded method.	… tens multiplied by … is equal to … tens. … ones multiplied by … is equal to … ones.  $20 \times 4 = 80$ $4 \times 4 = 16$ $24 \times 4 = 96$  	
Scaling Children focus on multiplication as scaling (…. times the size) as opposed to repeated addition.	There are …. times as many … as …  There are 3 times as many triangles as circles.	… is … times the size of … … is … times the length/height of …  Miss Smith is twice the height of Jo.

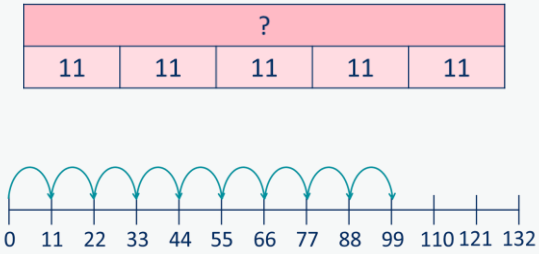
Multiplication

Progression of skills	Key representations
Correspondence problems (How many ways?) Encourage children to work systematically to find all the different possible combinations.	For every … , there are … possible ... There are …× … possibilities altogether.   For every hat, there are two possible scarves. $3 \times 2 = 6$ There are 6 possibilities altogether.

Multiplication

Year 4	• Recall multiplication facts for multiplication tables up to 12 × 12 • Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers. • Recognise and use factor pairs and commutativity in mental calculations. • Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. • Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.
Progression of skills	**Key representations**
Times-table facts to 12 × 12 Encourage daily counting in multiples both forwards and back. Encourage children to notice links between related times-tables.	… groups of … = … times … is equal to … … × … =
Multiply by 1 and 0	Any number multiplied by 1 is equal to … Any number multiplied by 0 is equal to …





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

… × … = …

$1 \times 1 = 1$ $1 \times 0 = 0$
$2 \times 1 = 2$ $2 \times 0 = 0$
$3 \times 1 = 3$ $3 \times 0 = 0$
$4 \times 1 = 4$ $4 \times 0 = 0$

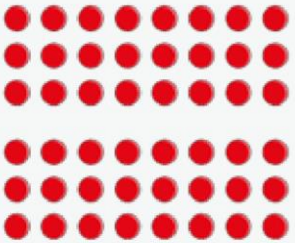
Multiplication

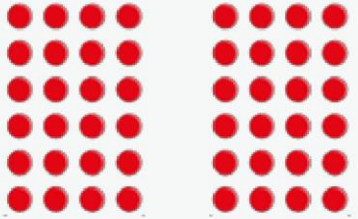
<table>
<tr><th>Progression of skills</th><th colspan="2">Key representations</th></tr>
<tr><td>Multiply 3 numbers

Children use their understanding of commutativity to multiply more efficiently.</td><td colspan="2">To work out … × … × …, I can first calculate … × … and then multiply the answer by …

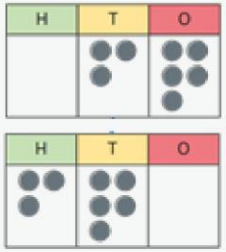
 
4 × 2 × 3 = 8 × 3 = 24
2 × 3 × 4 = 6 × 4 = 24
3 × 4 × 2 = 12 × 2 = 24</td></tr>
<tr><td>Factor pairs

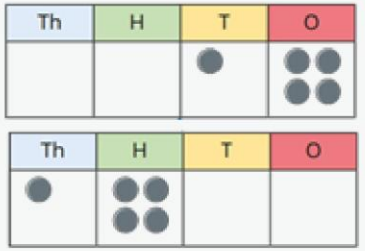
Children explore equivalent calculations using different factors pairs.</td><td colspan="2">12 = … × … , so … × 12 = … × … × …

 
8 × 6 = 8 × 3 × 2
8 × 6 = 24 × 2

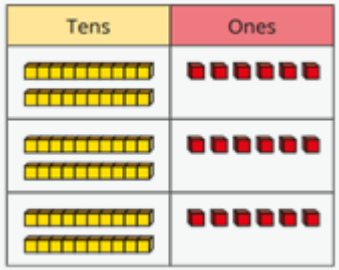
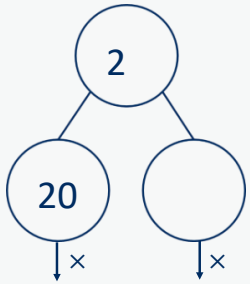
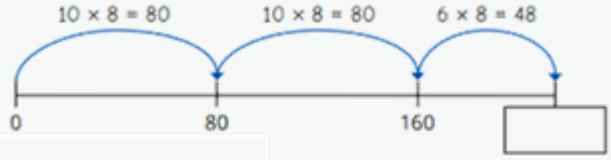
 
6 × 8 = 6 × 4 × 2
6 × 8 = 24 × 2</td></tr>
<tr><td>Multiply by 10 and 100

Some children may over-generalise that multiplying by 10 or 100 always results in adding zeros. This will cause issues later when multiplying decimals.</td><td>When I multiply by 10, the digits move … place value column to the left.
… is 10 times the size of …

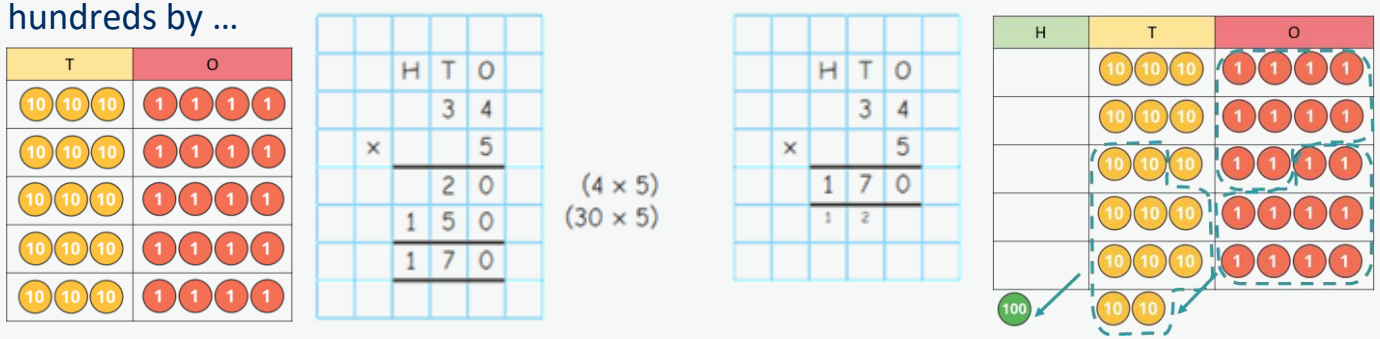
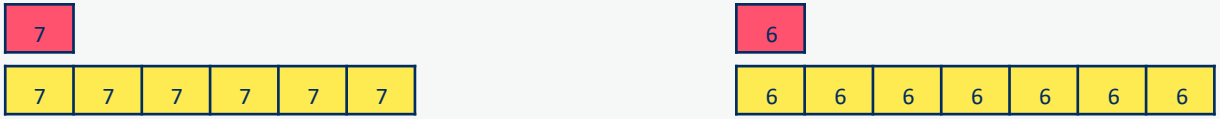
 
35 × 10 = 350</td><td>When I multiply by 100, the digits move … place value columns to the left.
… is 100 times the size of …

 
14 × 100 = 1,400</td></tr>
</table>

Multiplication

Progression of skills	Key representations
Related facts Use knowledge of multiplying by 10 and 100 to scale times-table facts.	… × … ones is equal to … ones so … × … tens is equal to … tens and … × … hundreds is equal to … hundreds.  $3 \times 7 = 21$ $3 \times 70 = 210$ $3 \times 700 = 2{,}100$ $7 \times 3 = 21$ $7 \times 30 = 210$ $7 \times 300 = 2{,}100$
Mental strategies Partition 2 or 3-digit numbers to multiply using informal methods.	… tens multiplied by … is equal to … tens. …ones multiplied by … is equal to … ones.  $3 \times 26 = 60 + 18 = 78$   $26 \times 8 = 80 + 80 + 48 = 208$

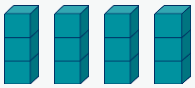
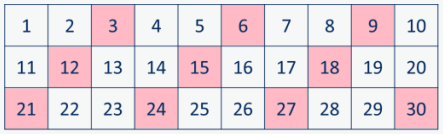
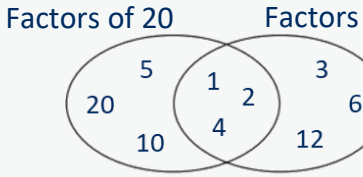
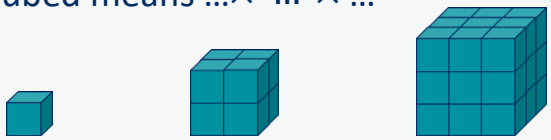
Multiplication

Progression of skills	Key representations
Multiply a 2 or 3-digit number by a 1-digit number The short multiplication method is introduced for the first time, initially in an expanded form.	To multiply a 2-digit number by … , I multiply the ones by … and the tens by … To multiply a 3-digit number by … , I multiply the ones by … , the tens by … and the hundreds by … 
Scaling Children focus on multiplication as scaling (… times the size).	… is … times the size of …  A computer mouse costs £7 A keyboard costs 6 times as much. A red ribbon is 6 cm. A yellow ribbon is 7 times as long.
Correspondence problems Encourage children to use tables to show all the different possible combinations.	For every … , there are … possibilities. There are …× … possibilities altogether. A pizza company offers a choice of 5 toppings and 3 bases. 5 × 3 = 15

	Deep pan	Italian	Thin
Cheese	C DP	C I	C Th
Mushroom	M DP	M I	M Th
Vegetable	V DP	V I	V Th
Chicken	C DP	C I	C Th
Tuna	T DP	T I	T Th

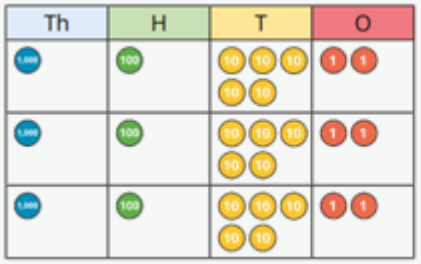
Multiplication

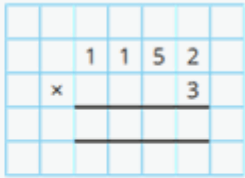
Year 5	• Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers • Recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) • Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers. • Multiply numbers mentally drawing upon known facts. • Multiply whole numbers and those involving decimals by 10, 100 and 1000 • Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

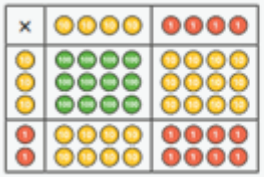
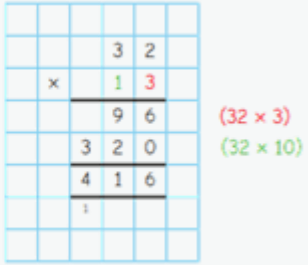
Progression of skills	Key representations		
Multiples and factors Encourage children to notice patterns and make links with known facts.	… is a multiple of … because … × … = …   1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30	… is a factor of … because … × … = …  1 × 8  2 × 4 1, 2, 4 and 8 are factors of 8	The common factors of … and … are … 
Square and cube numbers	… squared means … × …  1 × 1 = 1^2 = 1 2 × 2 = 2^2 = 4 3 × 3 = 3^2 = 9 4 × 4 = 4^2 = 16	… cubed means … × … × …  1 × 1 × 1 = 1^3 = 1 2 × 2 × 2 = 2^3 = 8 3 × 3 × 3 = 3^3 = 27	

Multiplication

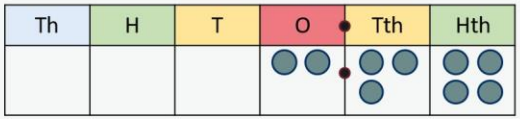
<table>
<tr><th>Progression of skills</th><th>Key representations</th></tr>
<tr><td>Multiply numbers up to 4 digits by a 1-digit number

This builds on the short multiplication method introduced in Y4</td><td colspan="2">To multiply a 4digit number by … , I multiply the ones by … , the tens by … , the hundreds by … and the thousands by …
 

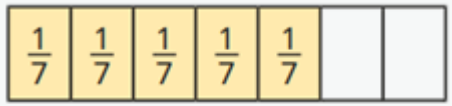
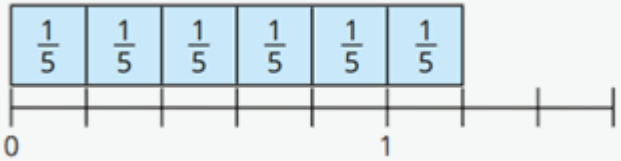
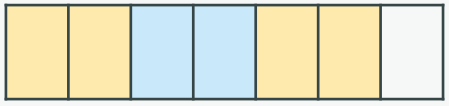
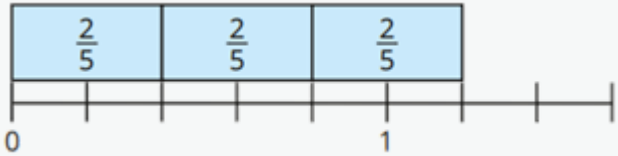
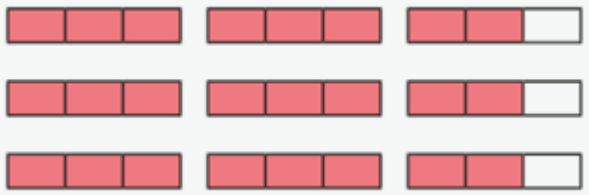

</td></tr>
<tr><td>Multiply numbers up to 4 digits by a 2-digit number

Numbers are first partitioned using an area model then long multiplication is introduced for the first time.</td><td>I can partition … into … and …
 
<table>
<tr><th>×</th><th>40</th><th>4</th></tr>
<tr><td>30</td><td>1,200</td><td>120</td></tr>
<tr><td>2</td><td>80</td><td>8</td></tr>
</table>
32 × 44 = 1,200 + 80 + 120 + 8
32 × 44 = 1,408</td><td>First, I multiply by the … Then I multiply by the …
<table>
<tr><th>×</th><th>10</th><th>3</th></tr>
<tr><td>30</td><td>300</td><td>90</td></tr>
<tr><td>2</td><td>20</td><td>6</td></tr>
</table>
300 + 90 + 20 + 6 = 416
 
</td></tr>
</table>

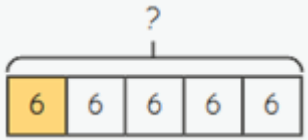
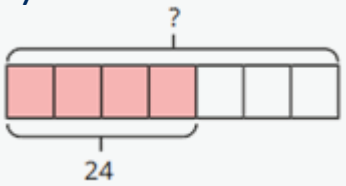
Multiplication

Progression of skills	Key representations
Multiply by 10, 100 and 1,000 Some children may over-generalise that multiplying by a power of 10 always results in adding zeros. This will cause issues later when multiplying decimals.	To multiply by 10/100/1,000, I move all the digits … places to the left. … is 10/100/1,000 times the size of …  234 × 10 = 2,340 234 × 100 = 23,400 234 × 1,000 = 234,000  2.34 × 10 = 23.4 2.34 × 100 = 234 2.34 × 1,000 = 2,340
Mental strategies Children continue to use efficient mental strategies such as partitioning and knowledge of factor pairs and related facts to multiply.	The most efficient strategy to calculate … × … is … To calculate … × 12, I can do … × … × … For example: 121 × 12 I could calculate 100 × 12 plus 20 × 12 plus 1 × 12 I could calculate 121 × 10 plus 121 × 2 I could calculate 121 × 6 × 2 I could calculate 121 × 4 × 3

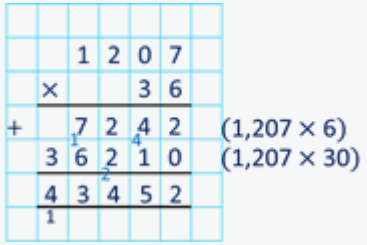
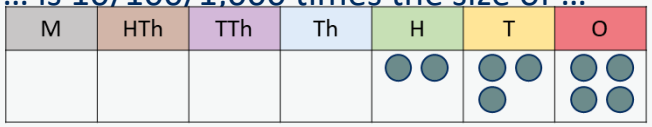
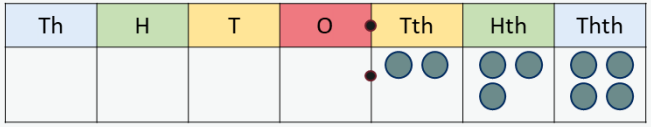
Multiplication

Progression of skills	Key representations
Multiply fractions by a whole number Make links withrepeated addition. E.g. $\frac{1}{}\times 4=\frac{1}{}+\frac{1}{}+\frac{1}{}+\frac{1}{}$	To multiply a fraction by an integer, I multiply the numerator by the integer and the denominator remains the same.  $\frac{1}{7}\times 5=\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{}{7}$  $\frac{1}{}\times 6=\quad=1\frac{1}{}$  $\frac{2}{7}\times 3=\frac{2}{7}+\frac{2}{7}+\frac{2}{7}=\frac{}{7}$  $\frac{2}{}\times 3=\quad=1\frac{1}{}$
Multiply mixed numbers by a whole number	I can partition $\frac{\square}{\square}$ into $\square$ and $\frac{\square}{\square}$  $2\frac{2}{}\times 3$ $2\times 3=6$ $\quad\frac{2}{}\times\quad=\quad=2$ $2\frac{2}{}\times 3=6+2=8$

Multiplication

Progression of skills	Key representations	
Find the whole Children multiply to find the whole from a given part.	If $\frac{1}{\square}$ is … , then the whole is …× …	If $\frac{\square}{\square}$ is … , then $\frac{1}{\square}$ is … and the whole is …× …
	$\frac{1}{5}$ of $\_\_ = 6$  $5 \times 6 = 30$ $\frac{1}{5}$ of **30** $= 6$	$\frac{3}{7}$ of $\_\_ = 24$  $\frac{1}{7} = 24 \div 4 = 6$ $7 \times 6 = 42$ $\frac{3}{7}$ of **42** $= 24$

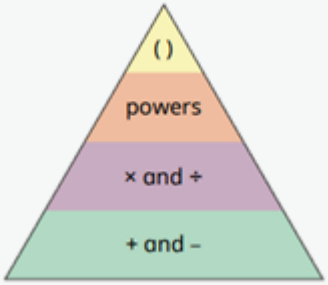
Multiplication

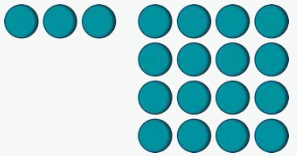
Year 6	• Identify common factors and common multiples. • Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. • Multiply numbers by 10, 100 and 1,000 • Multiply one-digit numbers with up to two decimal places by whole numbers. • Use their knowledge of the order of operations to carry out calculations involving the 4 operations. • Multiply simple pairs of proper fractions, writing the answer in its simplest form. • Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts. • Solve problems involving the calculation of percentages.
Progression of skills	**Key representations**
Multiply numbers up to 4 digits by a 2-digit number	To multiply by a 2-digit number, first multiply by the ones, then multiply by the tens and then find the total.  1 2 0 7 × 3 6 + 7 2 4 2 (1,207 × 6) 3 6 2 1 0 (1,207 × 30) 4 3 4 5 2
Multiply by 10, 100 and 1,000 Some children may over-generalise that multiplying by a power of 10 always results in adding zeros.	To multiply by 10/100/1,000, I move all the digits … places to the left. … is 10/100/1,000 times the size of …  M HTh TTh Th H T O  Th H T O Tth Hth Thth 234 × 10 = 2,340 234 × 100 = 23,400 234 × 1,000 = 234,000 0.234 × 10 = 2.34 0.234 × 100 = 23.4 0.234 × 1,000 = 234

Multiplication

<table>
<tr><th>Progression of skills</th><th>Key representations</th></tr>
<tr><td>Order of operations

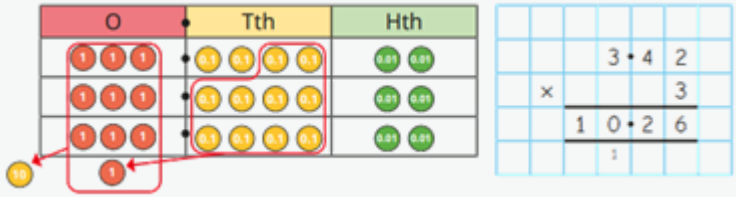
Calculations in brackets should be done first.
Multiplication and division should be performed before addition and subtraction.</td><td>… has greater priority than …, so the first part of the calculation I need to do is …

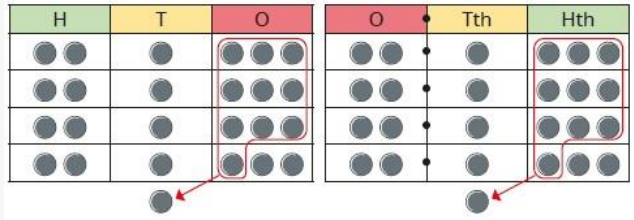
 


$(3 + 4) \times 2 = 14$
 
$3 + 4^2 = 19$
 
$3 + 4 \times 2 = 11$</td></tr>
<tr><td>Multiply decimals by integers

This is the first time children multiply decimals by numbers other than 10, 100 or 1,000
Encourage them to make links with known facts and whole number multiplication.</td><td>I know that … × … = …,
so I also know that… × … = …

 
$6 \times 2 = 12$
 
$6 \times 0.2 = 1.2$</td><td>I need to exchange 10 … for 1 …

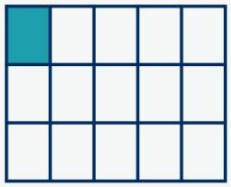


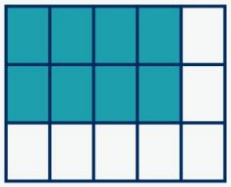
$213 \times 4 = 852$
$2.13 \times 4 = 8.52$</td></tr>
</table>

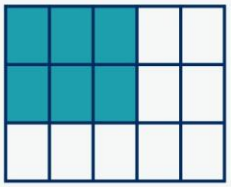
Multiplication

<table>
<tr><th>Progression of skills</th><th colspan="2">Key representations</th></tr>
<tr><td>Multiply fractions by fractions

Encourage children to give answers in their simplest form.</td><td colspan="2">When multiplying a pair of fractions, I need to multiply the numerator and multiply the denominator.

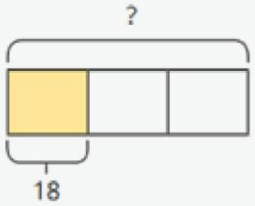
  $\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

  $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

  $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$</td></tr>
<tr><td>Find the whole

Children multiply to find the whole from a given part.</td><td>If $\frac{1}{\square}$ is … , then the whole is … × …

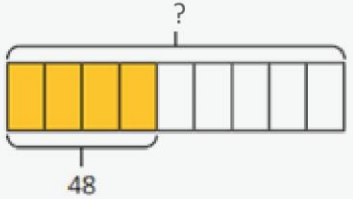
$\frac{1}{3}$ of ___ = 18

$18 \times 3 = 54$

$\frac{1}{3}$ of 54 = 18</td><td>If $\frac{\square}{\square}$ is … , then $\frac{1}{\square}$ is … and the whole is … × …

$\frac{4}{9}$ of ___ = 48

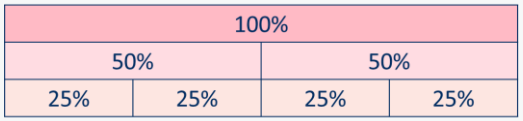
$\frac{1}{9} = 48 \div 4 = 12$

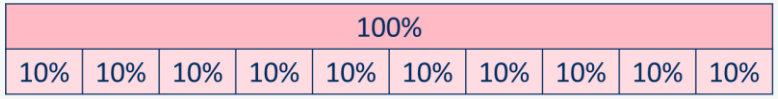
$9 \times 12 = 108$

$\frac{4}{9}$ of 108 = 48</td></tr>
</table>

Multiplication

<table>
<tr><th>Progression of skills</th><th colspan="2">Key representations</th></tr>
<tr><td>Calculate percentages

Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.</td><td>There are … lots of … % in 100%
To find … %, I need to divide by …
 

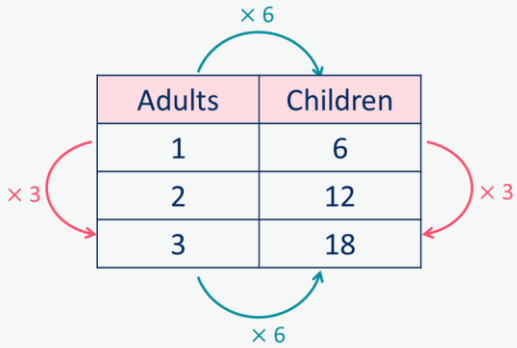
0% of … = …÷ 2
2 % of … = …÷ 4</td><td>… % is made up of … %, and … %
 

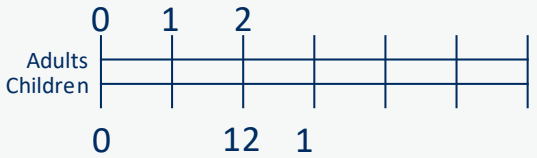
To find 30%, I can find 10% and then multiply it by 3
To find 23%, I can use 10% × 2 and 1% × 3
To find 99%, I can find 1%, then subtract from 100%</td></tr>
<tr><td>Calculations involving ratio

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent.
Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.</td><td colspan="2">For every … , there are …

For every 1 adult on a school trip, there are 6 children.

The ratio of adults to children is 1 : 6</td></tr>
</table>

Progression of skills - Division

Year group	Skill
Reception	• Sharing • Grouping
Year 1	• Make equal groups – grouping • Make equal groups – sharing • Find a half • Find a quarter

Progression of skills - Division

Year group	Skill
Year 2	• Divide by 2 • Divide by 10 • Divide by 5 • Missing numbers • Unit fractions • Non-unit fractions
Year 3	• Divide by 3 • Divide by 4 • Divide by 8 • Related facts • Divide a 2-digit number by a 1-digit number - no exchange • Divide a 2-digit number by a 1-digit number - with remainders • Unit fractions of a set of objects • Non-unit fractions of a set of objects

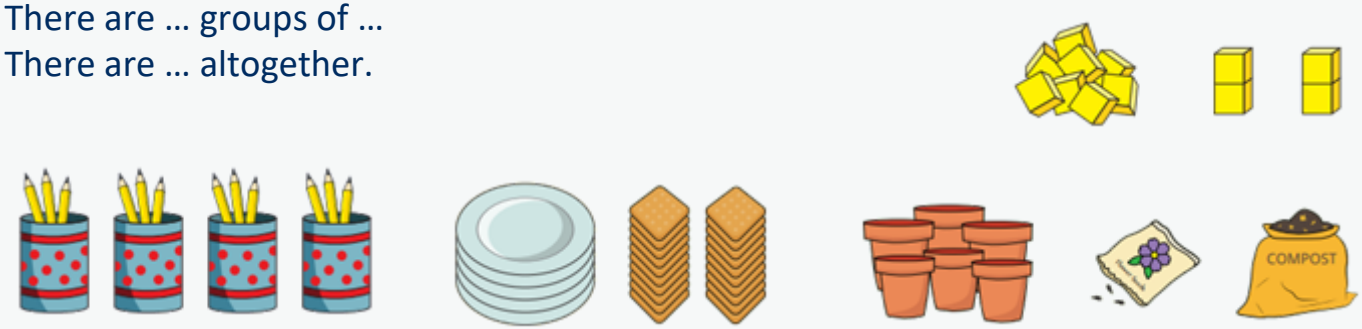
Progression of skills - Division

Year group	Skill
Year 4	• Division facts to 12 × 12 • Divide a number by 1 and itself • Related facts • Divide a 2 or 3-digit number by a 1-digit number • Divide by 10 and 100
Year 5	• Mental strategies • Divide numbers up to 4 digits by a 1-digit number • Divide by 10, 100 and 1,000 • Fraction of an amount

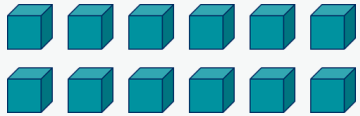
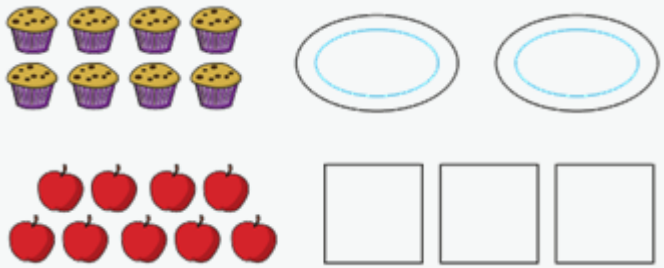
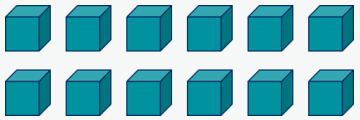
Progression of skills - Division

Year group	Skill
Year 6	• Short division • Mental strategies • Long division • Order of operations • Divide by 10, 100 and 1,000 • Divide decimals by integers • Decimal and fraction equivalents • Divide a fraction by an integer • Fraction of an amount • Calculate percentages • Calculations involving ratio

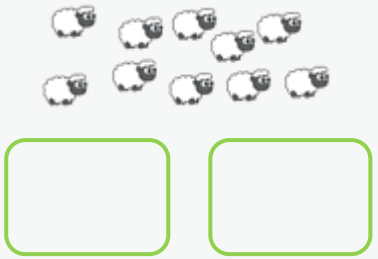
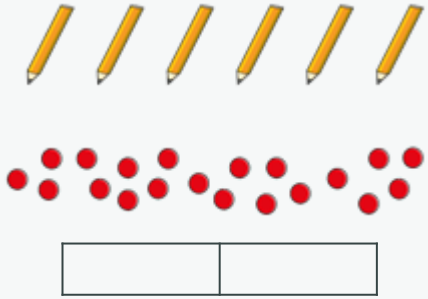
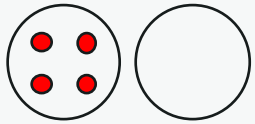
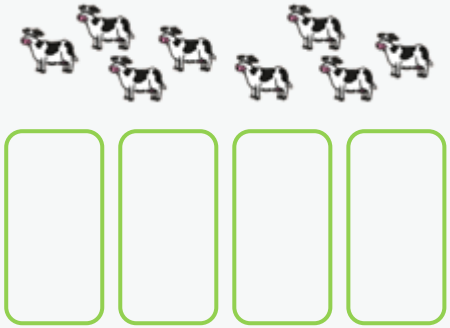
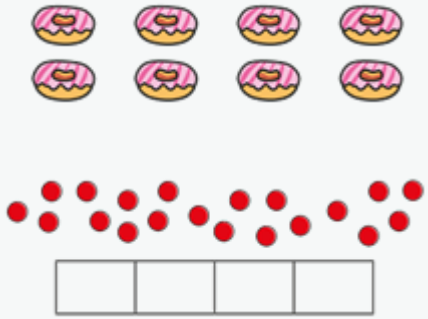
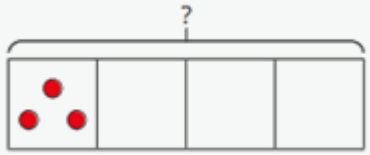
Division

Reception	• Have a deep understanding of number to 10, including the composition of each number. • Subitise (recognise quantities without counting) up to 5 • Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts. • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.
Progression of skills	**Key representations**
Sharing Provide practical activities such as sharing items during snack time. Encourage children to check whether items have been shared fairly (equally).	There are … altogether. They are shared equally between … groups. 
Grouping Provide opportunities to make equal groups when tidying up or during snack time. Encourage children to check that each group has the same amount.	There are … groups of … There are … altogether. 

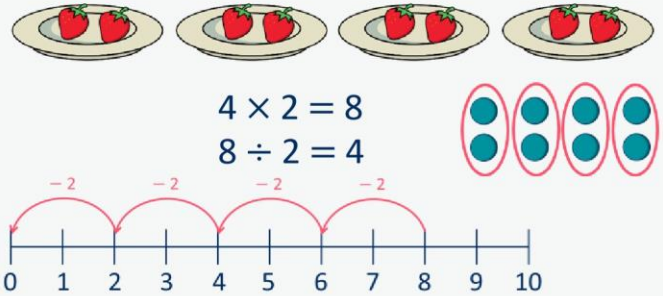
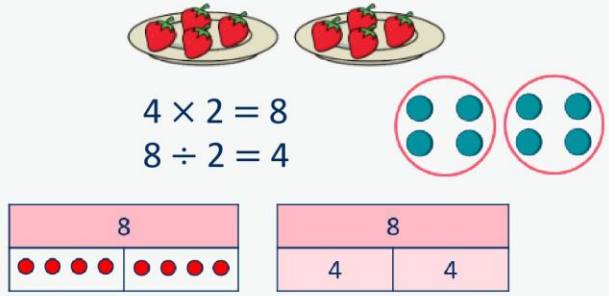
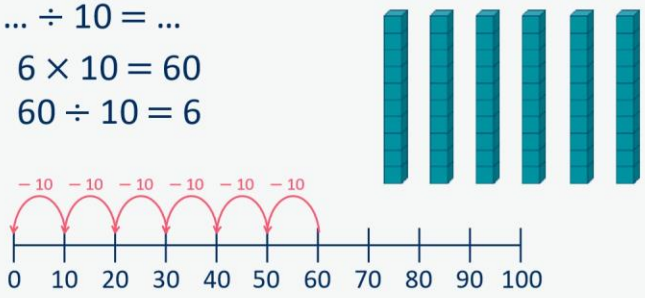
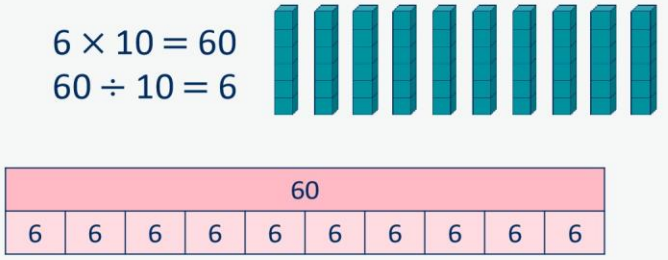
Division

Year 1	• Solve simple onestep problems involving division, using concrete objects, pictorial representations and arrays with the support of the teacher. • Recognise, find and name a half as one of two equal parts of a quantity. • Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.		
Progression of skills	**Key representations**		
Make equal groups– grouping Encourage children to physically move objects into equal groups. They can also circle equal groups when using pictures.	There are … altogether. How many groups of … can you make?  	Circle groups of 2 There are … groups of 2 	Take … cubes. Make equal groups.  There are … groups of …
Make equal groups– sharing Encourage children to check that the objects have been shared fairly and each group is the same.	… have been shared equally between… There are … on/in each … 		Take … cubes. Share them between …  12 shared between … is …

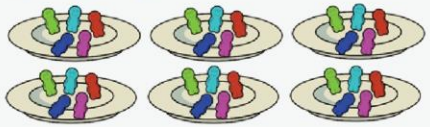
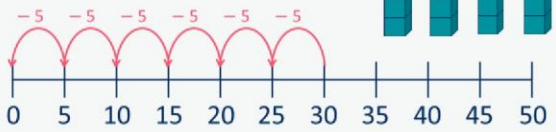
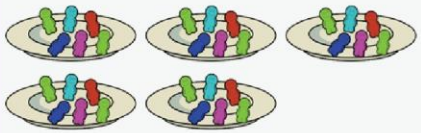
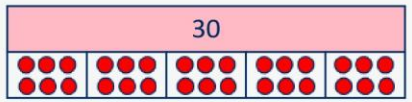
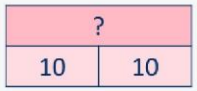
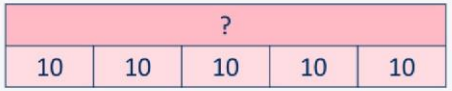
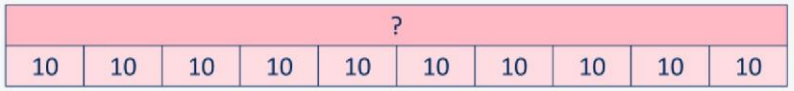
Division

Progression of skills	Key representations		
Find a half Start with practical opportunities to share a quantity into 2 groups. Progress to circling half of the objects in a picture and then to finding the whole from a given half.	To find half, I need to share into 2 equal groups.  There are … in each group.	Half of … is … 	If … is half, what is the whole?  is half of …
Find a quarter Start with practical opportunities to share a quantity into 4 groups. Progress to using pictures or bar models to find a quarter and then to finding the whole from a given quarter.	To find a quarter, I need to share into 4 equal groups.  There are … in each group.	A quarter of … is … 	If … is one quarter, what is the whole?  is one quarter of …

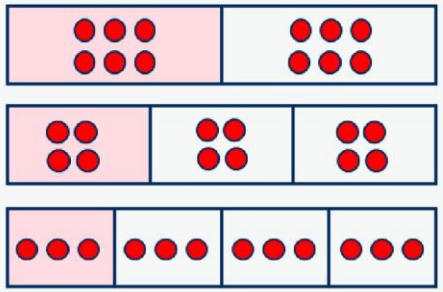
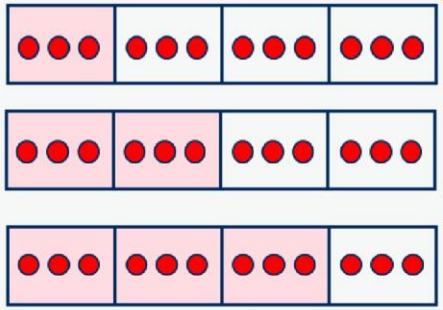
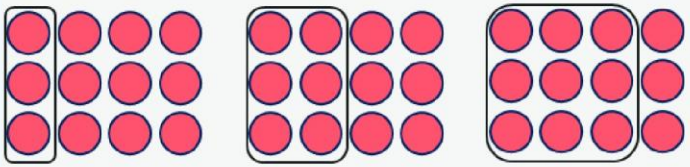
Division

Year 2	• Recall and use division facts for the 2, 5 and 10 multiplication tables. • Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs. • Recognise, find, name and write fractions $\frac{1}{3}, \frac{1}{4}, \frac{2}{4}$ and $\frac{3}{4}$ of a quantity.	
Progression of skills	**Key representations**	
Divide by 2 Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts and halving.	There are … equal groups of 2 … ÷ 2 = …  	… shared equally between 2 is … Half of … is … … ÷ 2 = … 
Divide by 10 Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are … equal groups of 10 … ÷ 10 = …  	… shared equally between 10 is … … ÷ 10 = … 

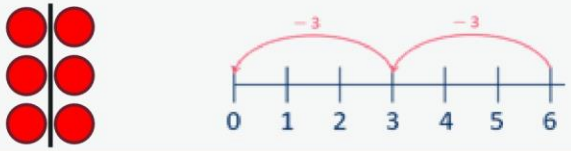
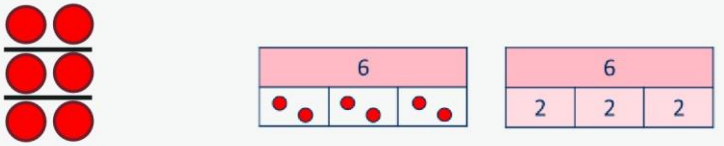
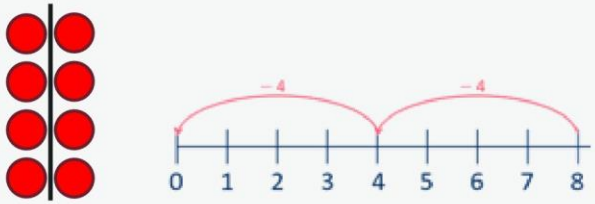
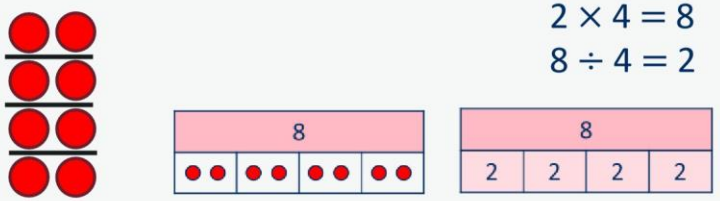
Division

Progression of skills	Key representations	
Divide by 5 Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are … equal groups of 5 … ÷ 5 = …  6 × 5 = 30 30 ÷ 5 = 6   	… shared equally between 5 is … … ÷ 5 = …  6 × 5 = 30 30 ÷ 5 = 6  
Missing numbers Bar models are useful to show the link between multiplication and division.	… divided by 2/5/10 is equal to …  □ ÷ 2 = 10  □ ÷ 5 = 10  □ ÷ 10 = 10	

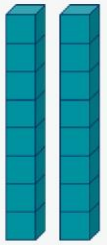
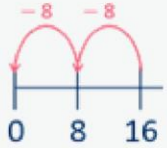
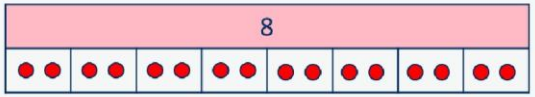
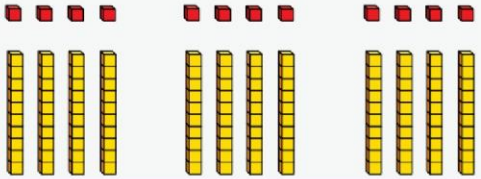
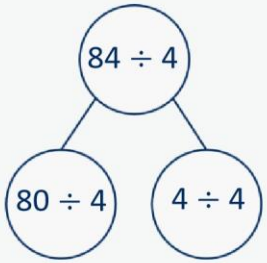
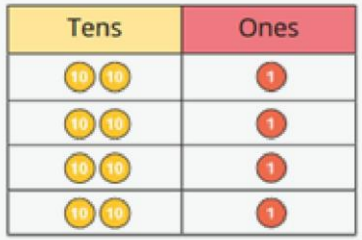
Division

Progression of skills	Key representations	
Unit fractions In Y2 the focus is on finding $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{3}$ Bar models are useful to show the link between division and finding a fraction.	The objects have been shared fairly into … groups. $\frac{1}{\square}$ of … is … 	There are … equal parts. There is … part circled. $\frac{1}{\square}$ is circled. 
Non-unit fractions In Y2 the focus is on finding $\frac{2}{4}$ and $\frac{3}{4}$ Prompt children to notice that $\frac{2}{4}$ is equivalent to $\frac{1}{2}$	The objects have been shared fairly into … groups. $\frac{\square}{\square}$ of … is … 	There are … equal parts. There are … parts circled. $\frac{\square}{\square}$ is circled. 

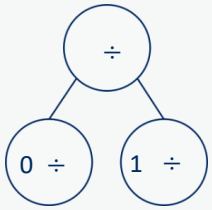
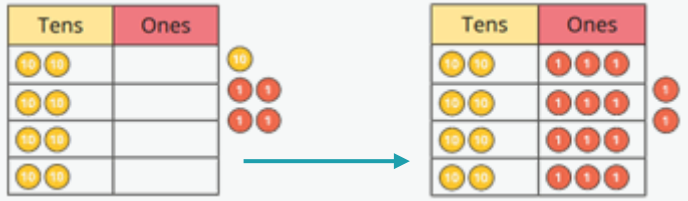
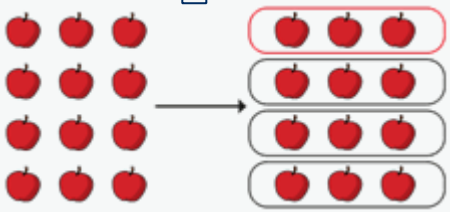
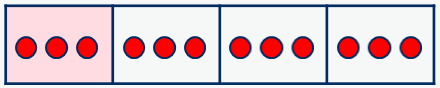
Division

Year 3	• Recall and use division facts for the 3, 4 and 8 multiplication tables. • Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. • Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.	
Progression of skills	**Key representations**	
Divide by 3 Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are … groups of 3 in … … ÷ 3 = $2 \times 3 = 6$ $6 \div 3 = 2$  	… has been shared equally into 3 equal groups. … ÷ 3 =  $2 \times 3 = 6$ $6 \div 3 = 2$ 
Divide by 4 Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are … groups of 4 in … … ÷ 4 =  $2 \times 4 = 8$ $8 \div 4 = 2$  	… has been shared equally into 4 equal groups. … ÷ 4 =  $2 \times 4 = 8$ $8 \div 4 = 2$ 

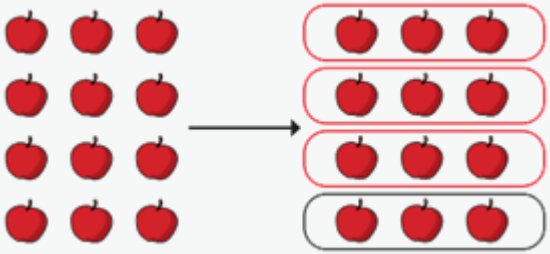
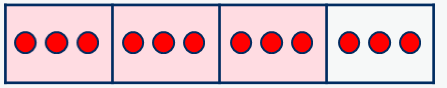
Division

Progression of skills	Key representations
Divide by 8 Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are … groups of 8 in … … ÷ 8 =  2 × 8 = 16 16 ÷ 8 = 2   … has been shared equally into 8 equal groups. … ÷ 8 =    2 × 8 = 16 16 ÷ 8 = 2
Related facts Link to known times-table facts.	… ÷ … is equal to …, so … tens ÷ … is equal to … tens.   12 ÷ 3 = 4 120 ÷ 3 = 40
Divide a 2-digit number by a 1-digit number - no exchange Partition into tens and ones to divide and then recombine.	… tens divided by … is equal to … tens. … ones divided by … is equal to … ones.  60 ÷ 2 = 30 4 ÷ 2 = 2 64 ÷ 2 = 32  

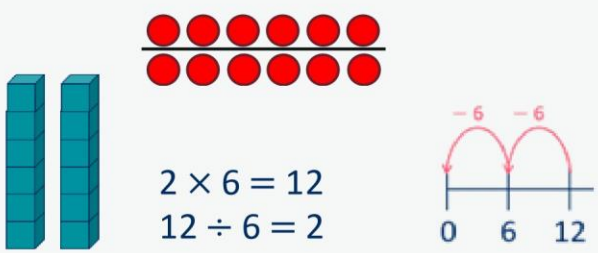
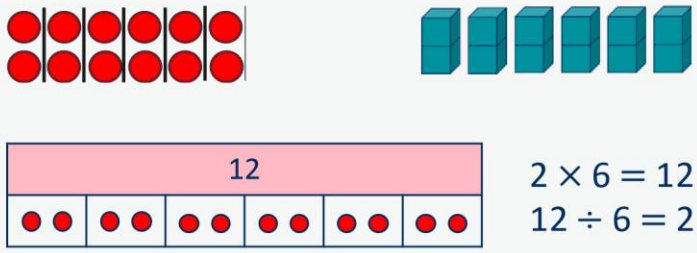
Division

Progression of skills	Key representations	
Divide a 2-digit number by a 1-digit number- with remainders Encourage children to partition numbers flexibly to help them to divide more efficiently.	… tens divided by … is equal to … tens. … ones divided by … is equal to … ones.   80 ÷ 4 = 20 16 ÷ 4 = 4 96 ÷ 4 = 24	There are … groups of … There are … remaining. 31 ÷ 4 = 7 r3  94 ÷ 4 = 23 r2 
Unit fractions of a set of objects Bar models are useful to show the link between division and fractions, for example, dividing by 3 and finding a third.	The whole is divided into … equal parts. Each part is $\frac{1}{\square}$ of the whole.  $\frac{1}{}$ of 12 apples is 3 apples.	ne … of … is … $\frac{1}{}$ of 12 is 3  $\frac{1}{}$ of 36 is 12 

Division

Progression of skills	Key representations	
Non-unit fractions of a set of objects Bar models are a useful representation and show the links with division and multiplication.	The whole is divided into … equal parts. Each part is $\frac{1}{\square}$ of the whole.  $\frac{1}{\square}$ of 12 apples is 9 apples.	$\frac{1}{\square}$ of … is …, so $\frac{\square}{\square}$ of … is … $\frac{1}{4}$ of 12 is 9  $\frac{2}{3}$ of 36 is 24 

Division

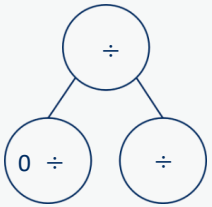
Year 4	• Recall division facts for multiplication tables up to 12 × 12 • Use place value, known and derived facts to divide mentally, including: dividing by 1 • Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.	
Progression of skills	**Key representations**	
Division facts to 12 × 12 Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	There are … groups of … in … … ÷ … =  	… has been shared equally into … equal groups. … ÷ … = 
Divide a number by 1 and itself Children may try to divide a number by zero and it should be highlighted that this is not possible.	When I divide a number by 1, the number remains the same. 5 shared between 1 is 5  There are **5** groups of 1 in 5 	When I divide a number by itself, the answer is 1 5 shared between 5 is 1  There is **1** group of 5 in 5 

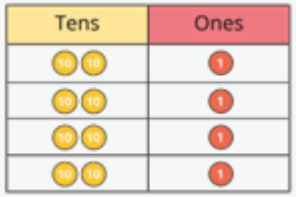
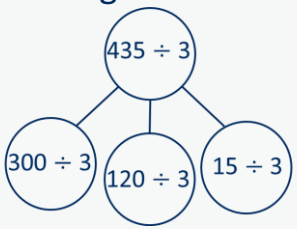
Division

<table>
<tr><th>Progression of skills</th><th>Key representations</th></tr>
<tr><td>Related facts

Link to known timestable facts.</td><td colspan="2">… ÷ … is equal to …
so … tens ÷ … is equal to … tens
and … hundreds ÷ … is equal to … hundreds.
 
21 ÷ 7 = 3
210 ÷ 7 = 30
2,100 ÷ 7 = 300

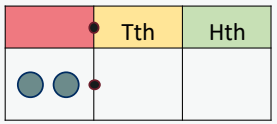
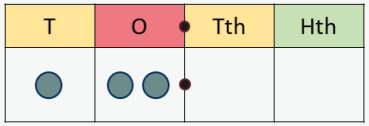
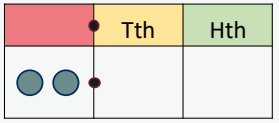
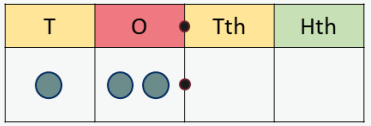
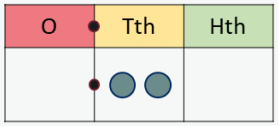
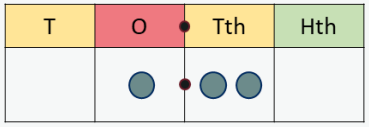
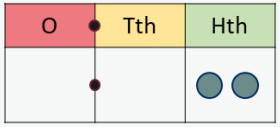
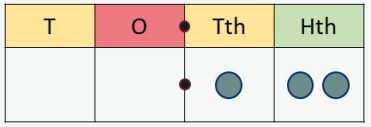
21 ÷ 3 = 7
210 ÷ 3 = 70
2,100 ÷ 3 = 700</td></tr>
<tr><td>Divide a 2 or 3-digit number by a 1-digit number

Progress from divisions with no exchange, to divisions with exchange and then divisions with remainders.</td><td>I can partition … into … tens and … ones.
 

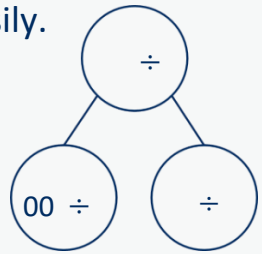
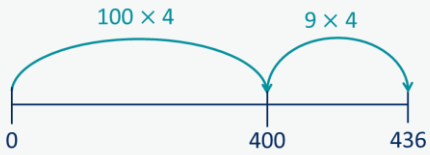
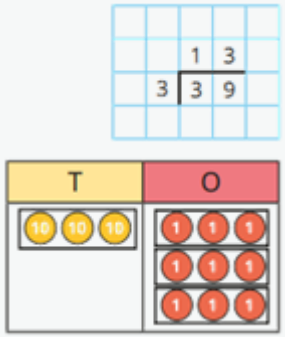
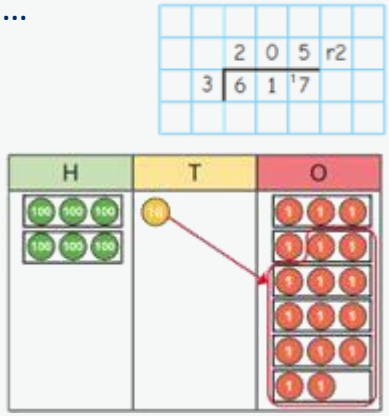
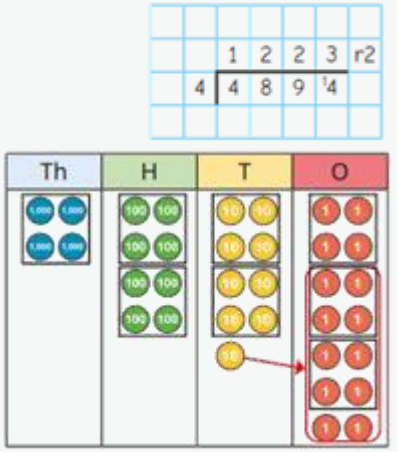
80 ÷ 4 = 20
4 ÷ 4 = 1
84 ÷ 4 = 21
 
</td><td>I cannot share the hundreds/tens equally, so I need to exchange 1 … for 10 …
 

300 ÷ 3 = 100
120 ÷ 3 = 40
15 ÷ 3 = 5
435 ÷ 3 = 145
 
</td></tr>
</table>

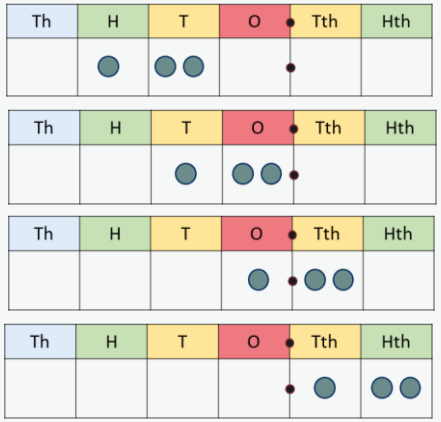
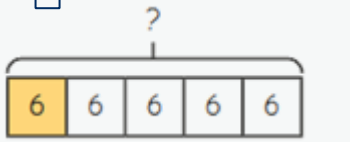
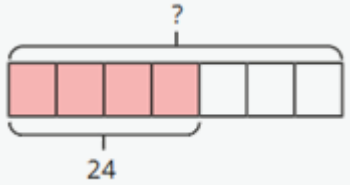
Division

Progression of skills	Key representations	
Divide by 10 and 100 Encourage children to notice that dividing by 100 is the same as dividing by 10 twice.	When I divide by 10, the digits move 1 place value column to the right. … is one-tenth the size of …	When I divide by 100, the digits move 2 place value columns to the right. … is one-hundredth the size of …
	 	 
	 	 
	$2 \div 10 = 0.2$ $12 \div 10 = 1.2$	$2 \div 100 = 0.02$ $12 \div 100 = 0.12$

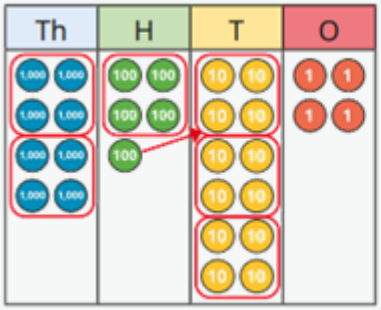
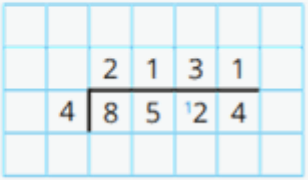
Division

Year 5	• Divide numbers mentally drawing upon known facts. • Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context. • Divide whole numbers and those involving decimals by 10, 100 and 1,000		
Progression of skills	**Key representations**		
Mental strategies	I can partition … into … and … to help me to divide more easily.  	I can show groups of … on a number line.  	To divide by …, I can divide by … and then divide the result by … 436 ÷ 4 = 436 ÷ 2 ÷ 2 436 ÷ 2 = 218 218 ÷ 2 = 109
Divide numbers up to 4 digits by a 1-digit number The short division method is introduced for the first time.	There are … groups of … hundreds/tens/ones/ in … I can exchange 1 … for 10 …    		

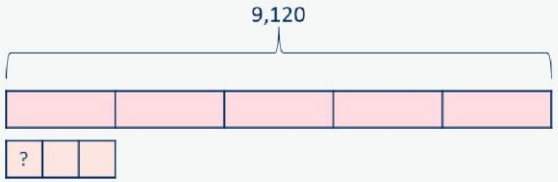
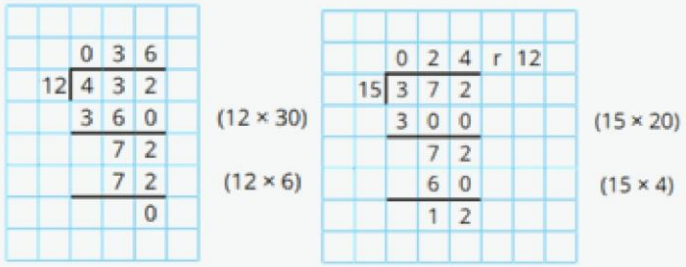
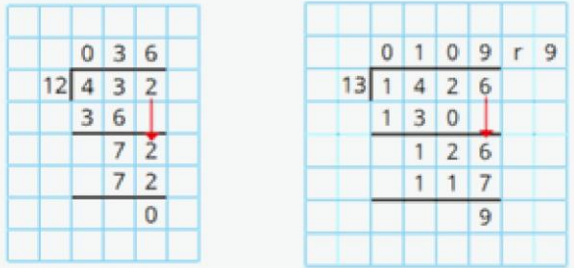
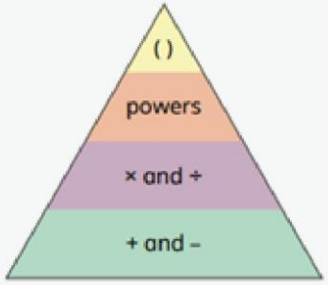
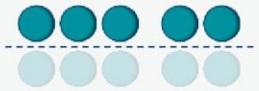
Division

Progression of skills	Key representations	
Divide by 10, 100 and 1,000 Encourage children to notice that dividing by 100 is the same as dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.	To divide by 10/100/1,000, I move all the digits … places to the right. … is one-tenth/one-hundredth/one-thousandth the size of … 	
Fraction of an amount Bar models support children to understand that to find a fraction of an amount, we divide by the denominator and multiply by the numerator.	To find $\frac{\square}{\square}$ of … , I need to divide by … and multiply by …  $\frac{1}{}$ of 20 = $\frac{}{}$ of 20 = $\frac{1}{}$ of = $\frac{}{}$ of =	If $\frac{1}{\square}$ is … , then the whole is …× …  $\frac{1}{}$ of ___ = 6  $\frac{}{7}$ of ___ = 24

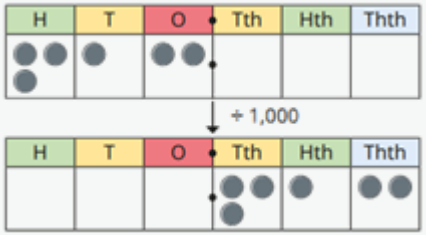
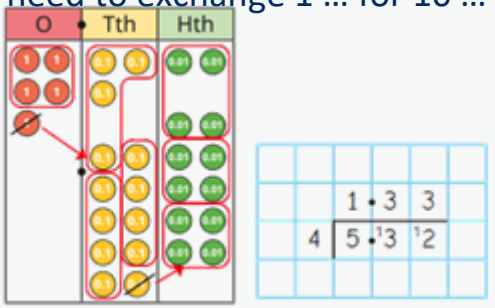
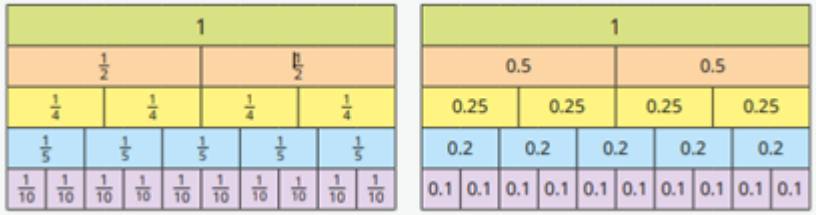
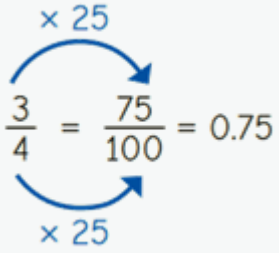
Division

Year 6	• Perform mental calculations, including with mixed operations and large numbers. • Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. • Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context. • Divide numbers by 10, 100 and 1,000 giving answers up to three decimal places. • Use written division methods in cases where the answer has up to two decimal places. • Associate a fraction with division and calculate decimal fraction equivalents. • Divide proper fractions by whole numbers [for example, $\frac{1}{2} \div 2 = \frac{1}{4}$] • Solve problems involving the calculation of percentages.
Progression of skills	**Key representations**
Short division Encourage children to interpret remainders in context, for example knowing that " remainder 1" could mean complete boxes with 1 left over so 5 boxes will be needed.	There are … groups of … hundreds/tens/ones/ in … I can exchange 1 … for 10 …  

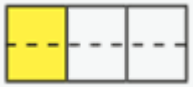
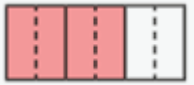
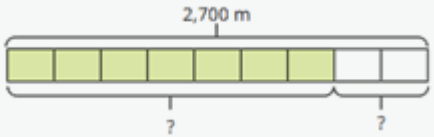
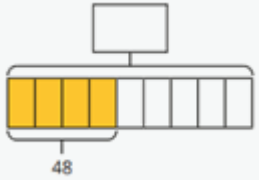
Division

Progression of skills	Key representations
Mental strategies Include partitioning and number line strategies outlined in Y5 as well as division using factors.	To divide by … , I can first divide by … and then divide the answer by … $240 \div 60 = 240 \div 10 \div 6$  $480 \div 24 = 480 \div 4 \div 6$  $9{,}120 \div 15 = 9{,}120 \div 5 \div 3$ 
Long division The long division method is introduced for the first time. Two alternative methods are shown.	Method 1  Method 2 
Order of operations Calculations in brackets should be done first, then powers. Multiplication and division should be performed before addition and subtraction.	… has greater priority than …, so the first part of the calculation I need to do is …   $(6 + 4) \div 2 = 5$  $6 + 4 \div 2 = 8$

Division

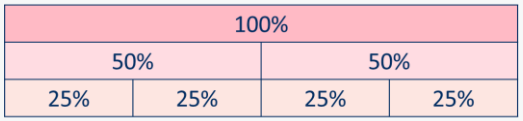
Progression of skills	Key representations	
Divide by 10, 100 and 1,000 Encourage children to notice that dividing by 100 is the same as dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.	To divide by … , I move the digits … places to the right.  312 ÷ 10 = 31.2 312 ÷ 100 = 3.12 312 ÷ 1,000 = 0.312 906 ÷ 10 = 90.6 906 ÷ 100 = 9.06 906 ÷ 1,000 = 0.906	
Divide decimals by integers This is the first time children divide decimals by numbers other than 10, 100 or 1,000	I know that … ÷ … = …, so I also know that… ÷ … = …  39 ÷ 3 = 13 3.9 ÷ 3 = 1.3 0.39 ÷ 3 = 0.13	I need to exchange 1 … for 10 … 
Decimal and fraction equivalents	The fraction … is equivalent to the decimal …  $\frac{1}{}$ = 0.2 $\frac{2}{}$ = 0.4 = 0.6	$\frac{\square}{\square}$ is equal to $\frac{\square}{100}$ 

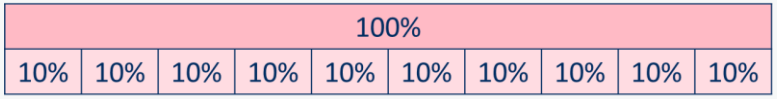
Division

Progression of skills	Key representations		
Divide a fraction by an integer This is the first time children divide fractions by an integer.	… ones divided by 2 is … ones so … sevenths divided by 2 is … sevenths.  $7 \div \quad = \frac{1}{7}$  $7 \div 2 = \frac{2}{7}$	I am dividing by … , so I can split each part into … equal parts.  $\frac{1}{\quad} \div 2 = \frac{1}{\quad}$	… is equivalent to … so … ÷ … = … ÷ …  $\frac{2}{\quad} =$ so $\frac{2}{\quad} \div \quad = \quad \div \quad = \frac{1}{\quad}$
Fraction of an amount Children divide and multiply to find fractions of an amount. Bar models can still be used to support understanding where needed.	To find $\frac{1}{\square}$ I divide by … $\frac{1}{2}$ of $\quad = \quad \div 2$ $\frac{1}{12}$ of $36 = \quad \div 12$	If $\frac{1}{\square}$ is equal to …, then $\square$ are equal to …  $\frac{7}{\quad}$ of $2{,}700 = \frac{1}{\quad}$ of $2{,}700 \times 7$	If $\frac{\square}{\square}$ is equal to …,then the whole is equal to …  of $\_\_ = 48$

Division

<table>
<tr><th>Progression of skills</th><th colspan="2">Key representations</th></tr>
<tr><td>Calculate percentages

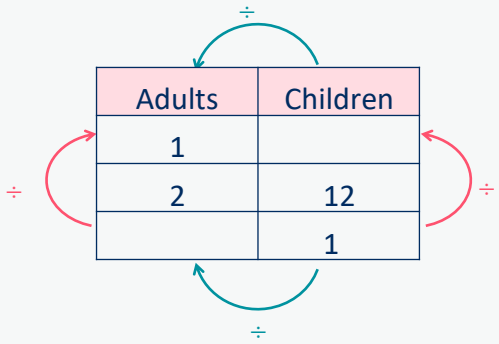
Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.</td><td>There are … lots of … % in 100%
To find … %, I need to divide by …
 

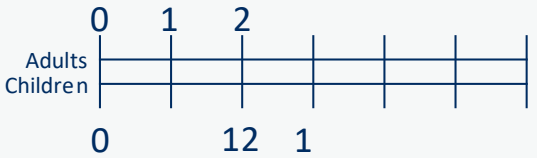
0% of … = …÷ 2
2 % of … = …÷ 4</td><td>… % is made up of … %, and … %
 

To find 30%, I can find 10% and then multiply it by 3
To find 23%, I can use 10% × 2 and 1% × 3
To find 99%, I can find 1%, then subtract from 100%</td></tr>
<tr><td>Calculations involving ratio

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent.
Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.</td><td colspan="2">For every … , there are …

For every 6 children on a school trip, there is 1 adult.
 

The ratio of children to adults is 6 : 1</td></tr>
</table>